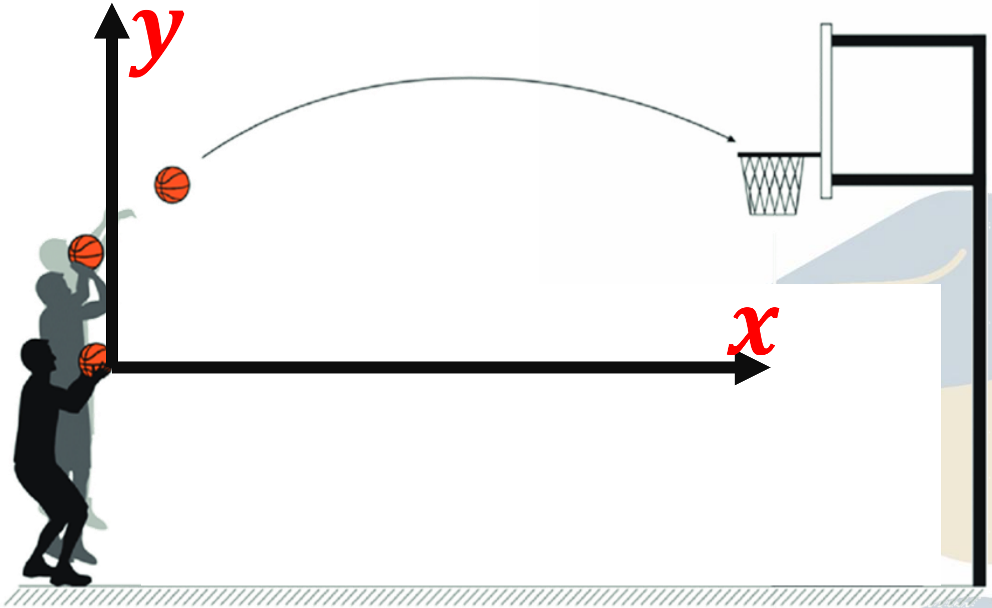


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Revision about vectors

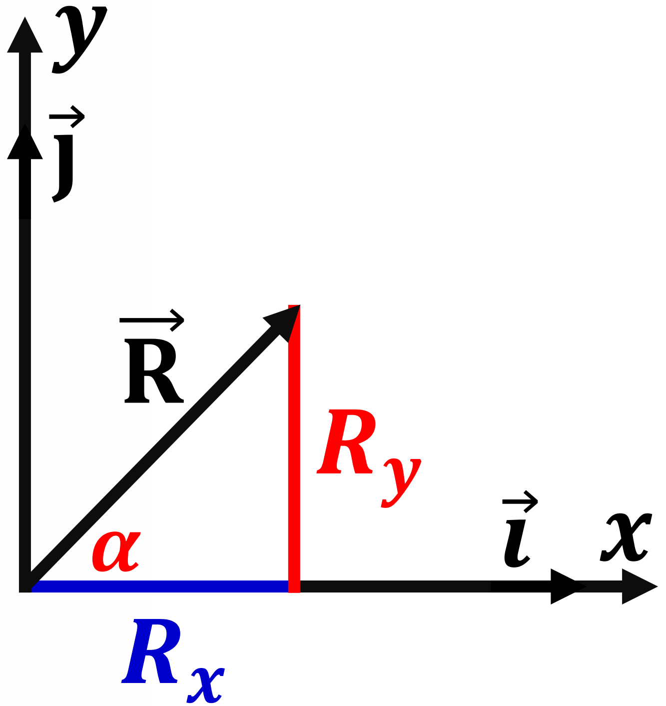
In 2 dimensions, a vector can be expressed in different forms.

In Cartesian form a vector (R) is composed of two components: R_x and R_y.

It is written as: $\vec{R} = R_x.\vec{i} + R_y.\vec{j}$

$$\tan(\alpha) = \frac{R_y}{R_x}$$

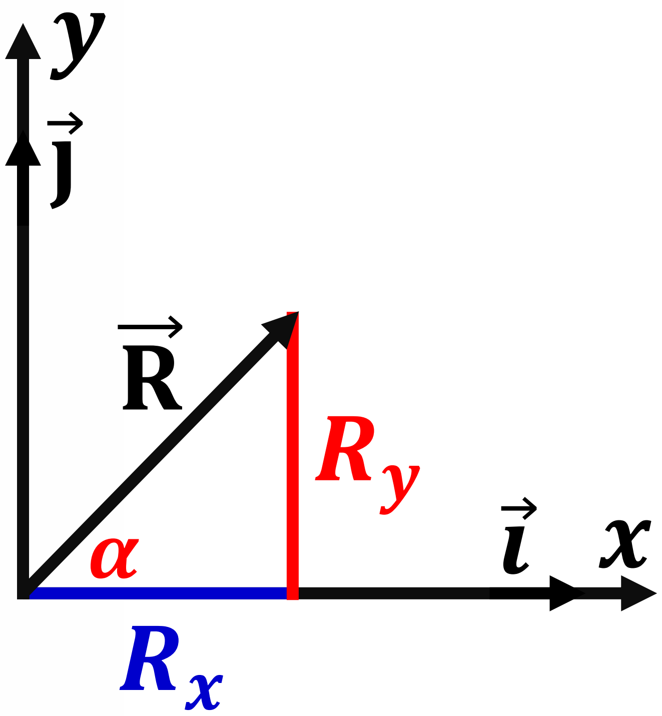
The magnitude is $R = \sqrt{R_x^2 + R_y^2}$



Revision about vectors

Application 1: Consider the vector $\vec{R} = 16\vec{i} + 15\,\vec{j}$

1)Determine the magnitude of the vector $\vec{R}$.



$$R = \sqrt{R_x{}^2 + R_y{}^2} \Rightarrow R = \sqrt{16^2 + 15^2}$$

$$R = 21.9$$

2)Deduce the value of the angle α.

$$\tan\alpha = \frac{R_y}{R_x} = \frac{15}{16} \Rightarrow \tan\alpha = 0.93 \Rightarrow \alpha = tan^{-1}(0.93)$$

$$\alpha = 43.2^\circ$$

Revision about vectors

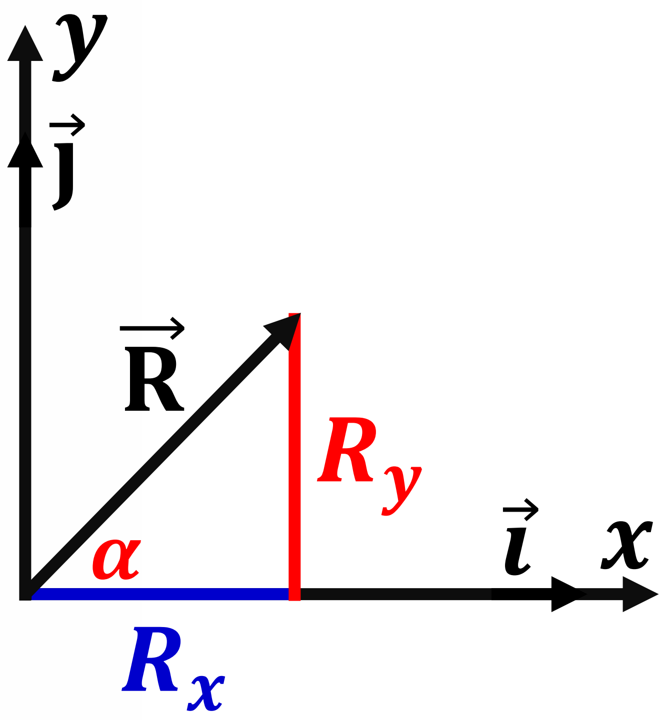
The components R_x and R_y can be calculated by:

$$sin(\alpha) = \frac{opp}{hyp} \Rightarrow sin(\alpha) = \frac{R_y}{R}$$

$$R_y = R.sin(\alpha)$$

$$cos(\alpha) = \frac{adj}{hyp} \Rightarrow cos(\alpha) = \frac{R_x}{R}$$

$$R_x = R.cos(\alpha)$$



Revision about vectors

Application 2:

Consider a velocity vector with a magnitude $V = 95Km/h$ as shown in the figure.

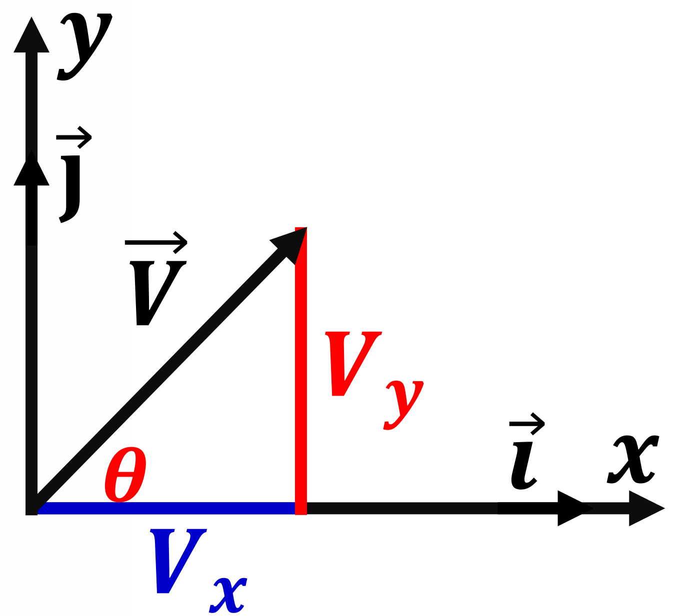
The velocity vector makes an angle $\theta = 20^{\circ}$ with the x-axis.

1)Determine the components of velocity vector.

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos\theta = \frac{V_x}{V}$$

$$V_x = V.\cos\theta \Rightarrow V_x = 95\cos 20^{\circ}$$

$$V_x = 89\ \text{km/h}$$

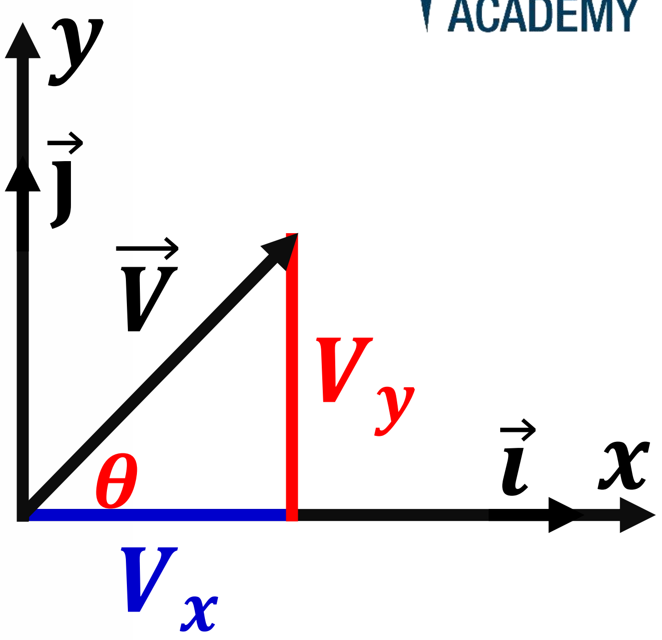


Revision about vectors

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin\theta = \frac{v_y}{V}$$

$$v_y = V.\sin\theta \Rightarrow v_y = 95\sin 20°$$

$$v_y = 32\ \text{km/h}$$



2)Write the velocity vector in cartesian form.

$$\vec{V} = V_x\vec{i} + V_y\vec{j}$$

$$\vec{V} = 89\,\vec{i} + 32\,\vec{j}$$

Revision about vectors

Addition of vectors:

$$\vec{U} = U_x\vec{i} + U_y\vec{j}$$

$$\vec{V} = V_x\vec{i} + V_y\vec{j}$$

$$\vec{W} = \vec{U} + \vec{V} \Rightarrow \vec{W} = (U_x + V_x)\vec{i} + (U_y + V_y)\vec{j}$$

$$\vec{T} = \vec{U} - \vec{V} \Rightarrow \vec{T} = (U_x - V_x)\vec{i} + (U_y - V_y)\vec{j}$$

Revision about vectors

Application 3:

Consider the two vectors $\vec{U} = 7\vec{i} + 8\vec{j}$ and $\vec{V} = 10\vec{i} - 6\vec{j}$.

1)Determine the vector $\vec{W}$, where $\vec{W} = \vec{U} + \vec{V}$.

$$\vec{W} = \vec{U} + \vec{V}$$

$$\vec{W} = (U_x + V_x)\vec{i} + (U_y + V_y)\vec{j}$$

$$\vec{W} = (7 + 10)\vec{i} + (8 - 6)\vec{j}$$

$$\vec{W} = 17\vec{i} + 2\vec{j}$$

Revision about vectors

$\vec{U} = 7\vec{i} + 8\vec{j}$ and $\vec{V} = 10\vec{i} - 6\vec{j}$.

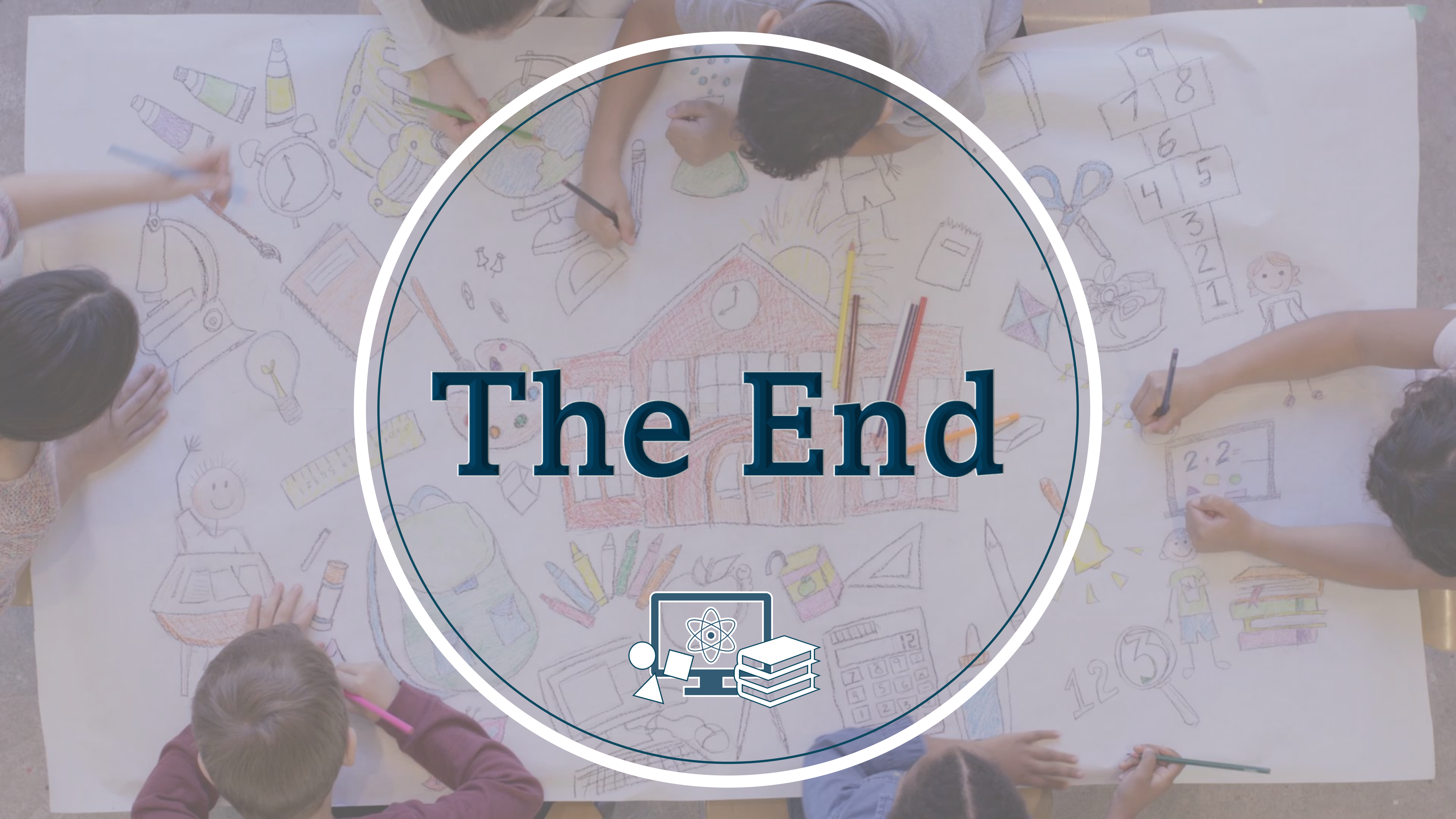
2)Determine the vector $\vec{R}$, where $\vec{R} = \vec{U} - \vec{V}$.

$$\vec{R} = \vec{U} - \vec{V}$$

$$\vec{R} = (U_x - V_x)\vec{i} + (U_y - V_y)\vec{j}$$

$$\vec{R} = (7 - 10)\vec{i} + (8 - (-6))\vec{j}$$

$$\vec{R} = -3\vec{i} + +14\vec{j}$$

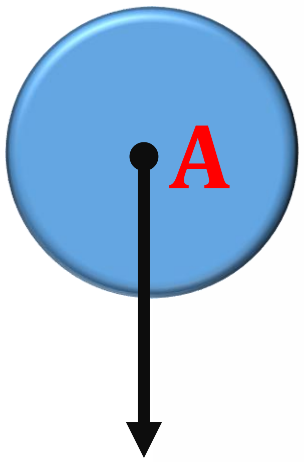


The End

Characteristics of a vector

Point of application:

Point of application is the point where the vector is applied.



The point of application is the point A

The point of application is the point M.

Characteristics of a vector

Line of action:

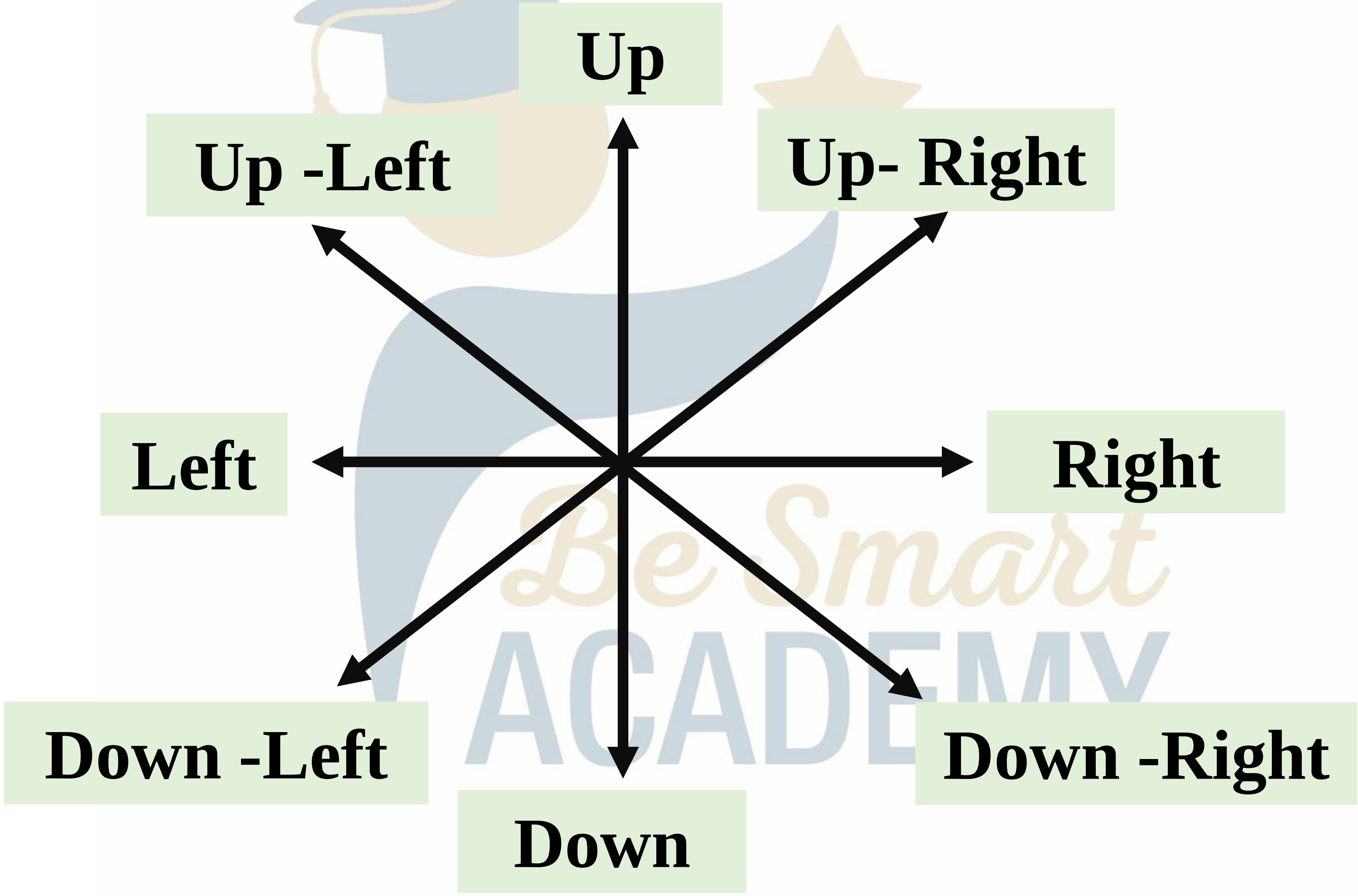
Line of action is the line that contain the applied vector

Line of action

- **Horizontal**
- **Vertical**
- **Oblique**

Characteristics of a vector

Direction: it refers to how the force is directed:

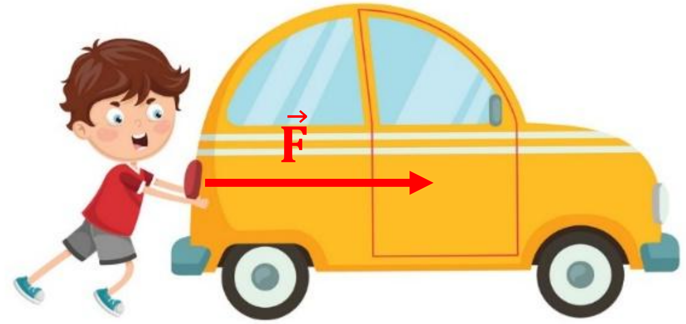


Characteristics of a vector

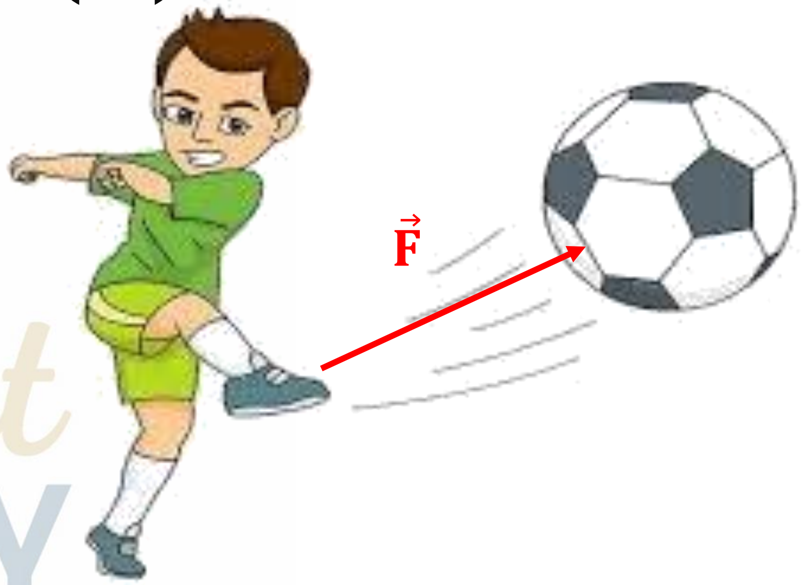
Magnitude:

Magnitude is the value of the applied vector.

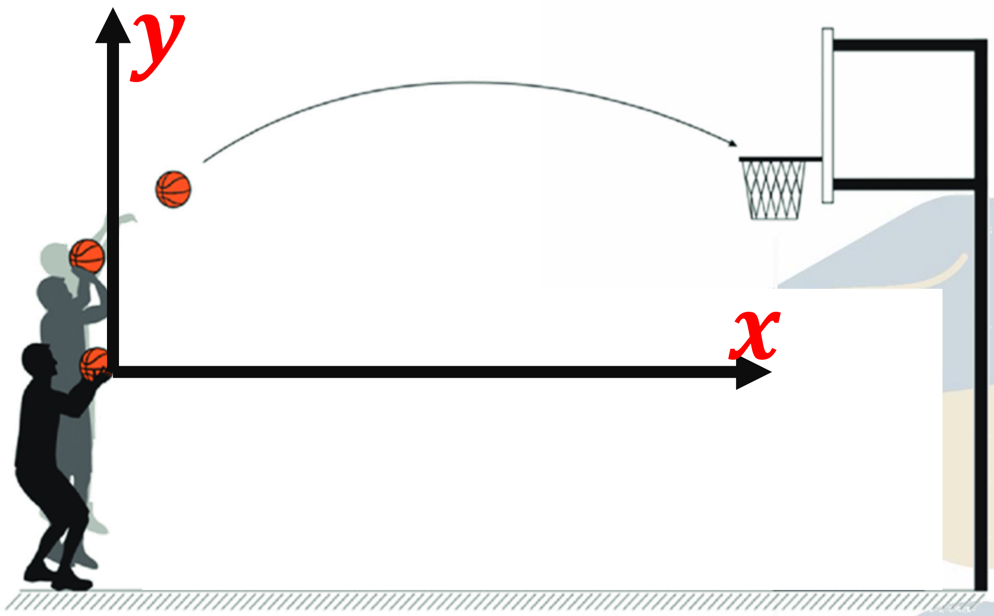
For force vector, it is expressed in Newton (N)



A boy pushes a car by a force of magnitude F=120N



A player kicks the ball by a force F=200N



Physics - Grade 11 S

Unit Two: Mechanics

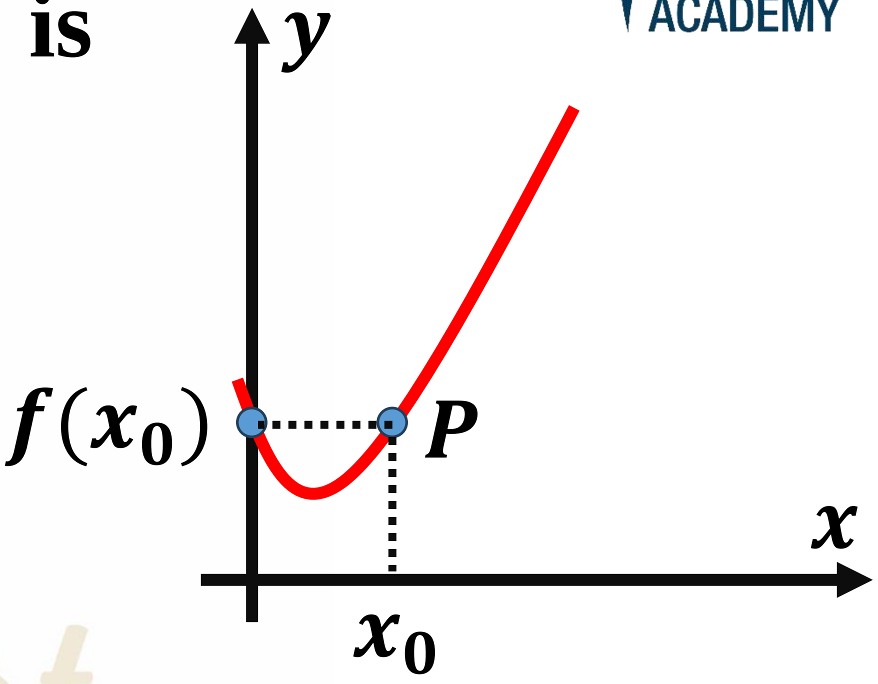
Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Basic rules of the derivative

The derivative of a function f(x) w.r.t x is the rate of change of y relative to x.



The derivative of a function f(x) is denoted by:

$$\frac{df(x)}{dx} \quad \text{Or} \quad f'(x)$$

In physics, the functions are variable in time (t):

The derivative of a function f(t) is denoted by:

$$\frac{df(t)}{dt} = f'(t)$$

Basic rules of the derivative

Functions f(t)	Derivative $f'(t)$	Examples	Derivative $f'(t)$
$f(t) = a$	$f'(t) = 0$	$f(t) = 2$	$f'(t) = 0$
$f(t) = at$	$f'(t) = a$	$f(t) = 5t$	$f'(t) = 5$
$f(t) = at^n$	$f'(t) = nat^{n-1}$	$f(t) = 7t^3$	$f'(t) = 7 \times 3t^{3-1}$ $f'(t) = 21t^2$

Basic rules of the derivative

$f(t) = u.v$	$f'(t) = u'v + v'u$

$$f(t) = (2t - 1)(t^3 + 4t)$$

$u = 2t - 1 \Rightarrow u' = 2$

$v = t^3 + 4t \Rightarrow V' = 3t^2 + 4$

$$f'(t) = u'v + v'u$$

$$f'(t) = 2\left(t^3 + 4t\right) + (3t^2 + 4)(2t - 1)$$

Basic rules of the derivative

$f(t) = \dfrac{u}{v}$	$f'(t) = \dfrac{u'v - v'u}{v^2}$

$$f(t) = \frac{2t - 1}{t^3 + 4t}$$

$u = 2t - 1$ → $u' = 2$

$v = t^3 + 4t$ → $v' = 3t^2 + 4$

$$f'(t) = \frac{2\left(t^3 + 4t\right) - (3t^2 + 4)(2t - 1)}{(t^3 + 4t)^2}$$

Basic rules of the derivative

$f(t)=\sqrt{u}$	$f'(x)=\dfrac{u'}{2\sqrt{u}}$

$$f(t)=f=\sqrt{2t^3}$$

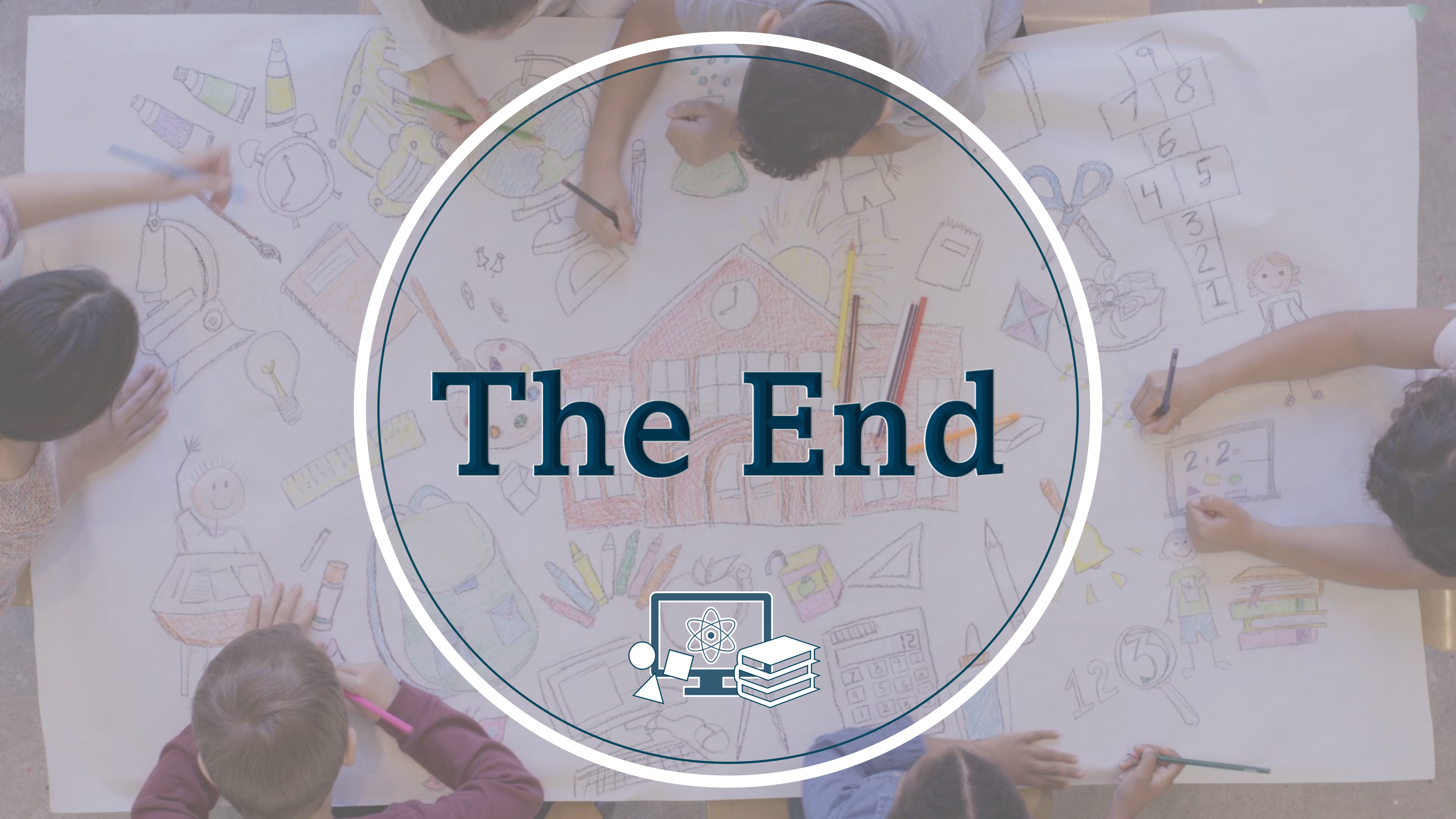
$u=2t^3$ ⇨ $u'=6t^2$

$f'(t)=\dfrac{u'}{2\sqrt{u}}$ ⇨ $f'(t)=\dfrac{6t^2}{2\sqrt{2t^3}}$

$$f'(t)=\frac{3t^2}{\sqrt{2t^3}}$$

Basic rules of the derivative

Functions f(t)	Derivative f'(t)	Examples
$f(t) = \sin t$	$f'(t) = \cos t$	
$f(t) = \sin u$	$f'(t) = u'cosu$	$f(t) = \sin(3t - 4)$ $f'(t) = 3\cos(3t - 4)$
$f(t) = \cos t$	$f'(t) = -sint$	
$f(t) = \cos u$	$f'(u) = -u'sinu$	$f(t) = \cos(3t - 4)$ $f'(t) = -3\sin(3t - 4)$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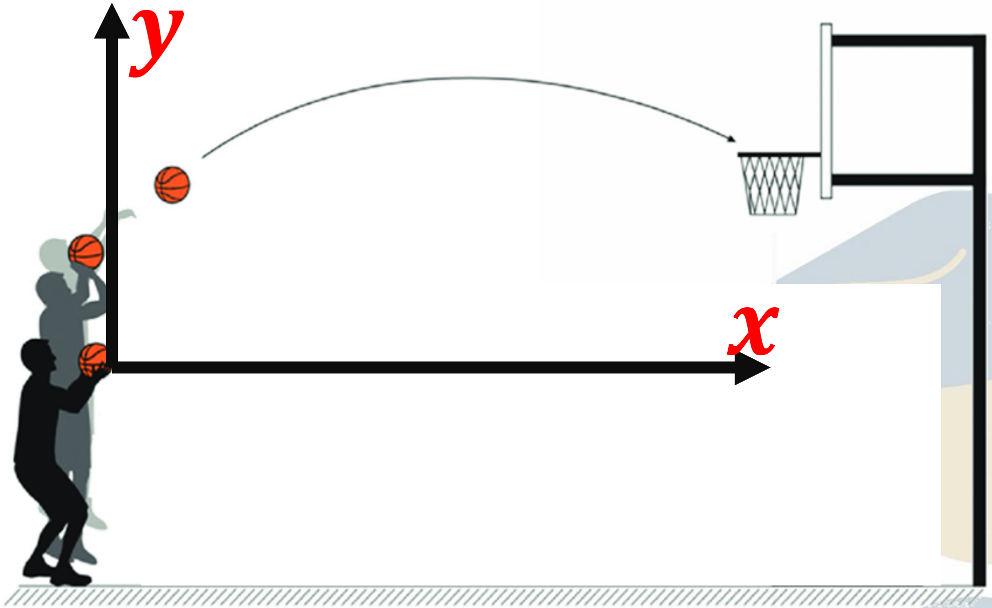


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

1. Difference between kinematics and dynamics
2. Types of motion
3. Why to study this lesson and where we use it?

Difference between kinematics and dynamics

Kinematics	**Kinematics is the branch of mechanics that studies the motion of objects (position, the velocity and the acceleration) without studying its causes (forces).**
Dynamics	**Dynamics is a branch of mechanics that studies the motion of the object by studying its causes (Forces)**

In this lesson we will study Kinematics

Types of motion

- Motion
  - Oscillatory motion
  - Translational motion
    - Circular motion
    - Curvilinear motion
    - Rectilinear motion
      - URM
      - UARM
      - UDRM
  - Rotational motion
  - combined motion

In this lesson we will study translational motion

Why to study this lesson? Where we use it?

- **We study kinematics to develop a method to describe and explain the motion of real-world objects.**
- **When describing the motion of an object, we use words such as going fast, stopped, slowing down, speeding up …**
- **We will be expanding upon this vocabulary list with words such as: Position, displacement, Speed, velocity and Acceleration.**

Why to study this lesson? Where we use it?

In our daily life, if an object is in motion, we may ask the following questions :

1. What is the shape of the path described by the object during its motion ?
2. What is the position of the body in motion after 5 sec, 10 sec, ..
3. What is its speed at a certain instant?

Why to study this lesson? Where we use it?

4. What is the **acceleration** of this object at 4 s, 7s ?

5. What is the **angular position** of the person after 5 sec, 10 sec, ..

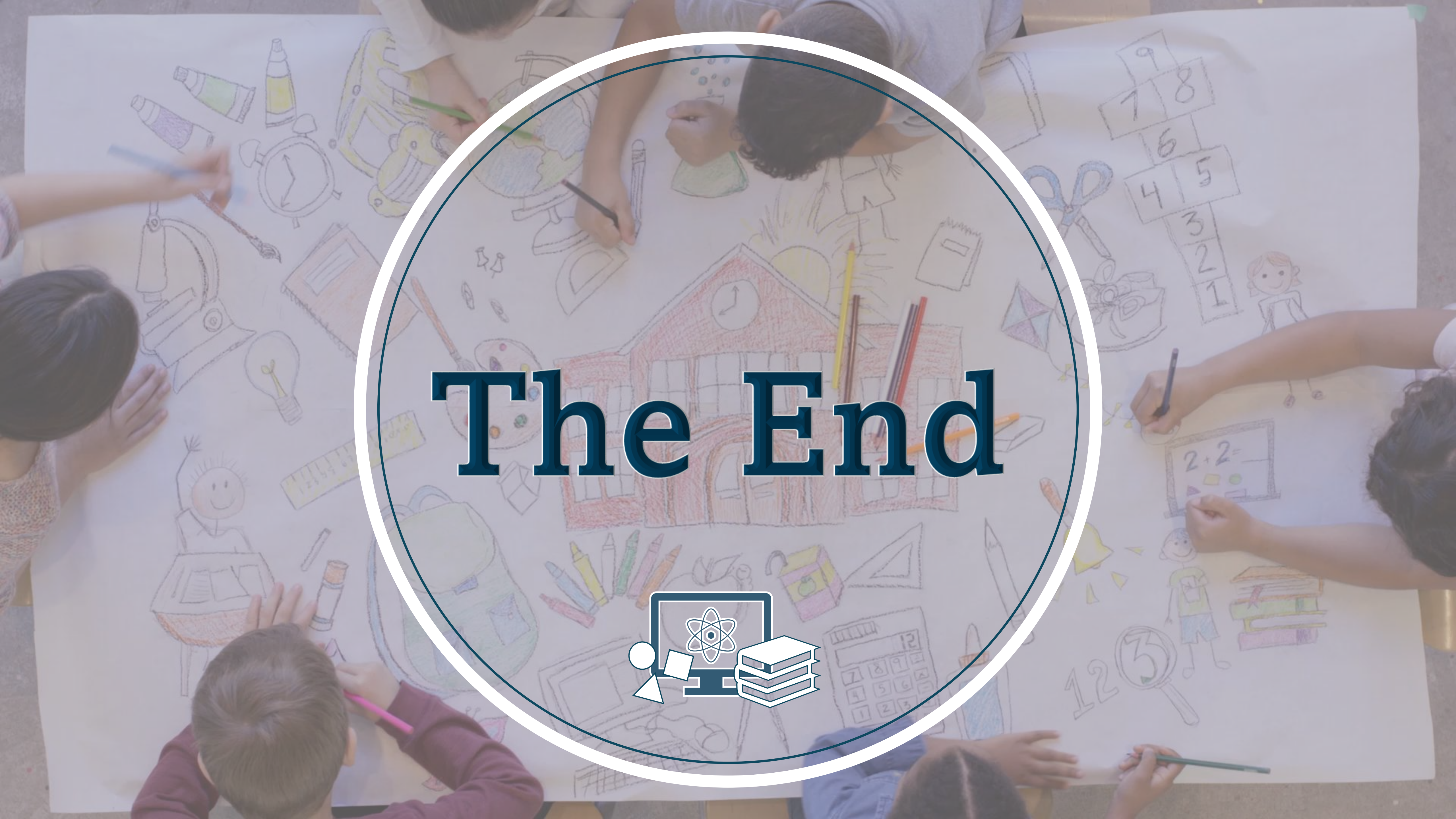
By answering these questions, we will be able to study the motion of any object in real life such as : tennis ball, motorcycle…

Reference frame (time and space)

Reference frame is a system of an origin (time and space) and axes with respect to which motion is to be studied.

We choose a coordinate system xoy (O, $\vec{i}$, $\vec{j}$) for the plan motion.

It is useful to we choose $t = 0$ as an origin of time.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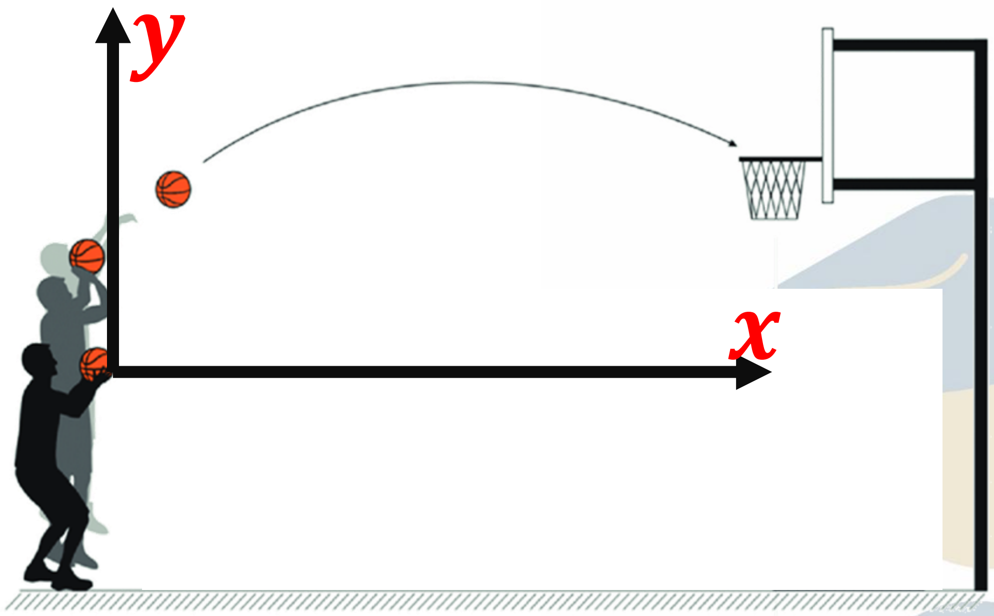


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

1. Determine the position vector and its magnitude
2. Determine the displacement vector and its magnitude

The position

Consider a particle M is in motion in a plane on a curve C.

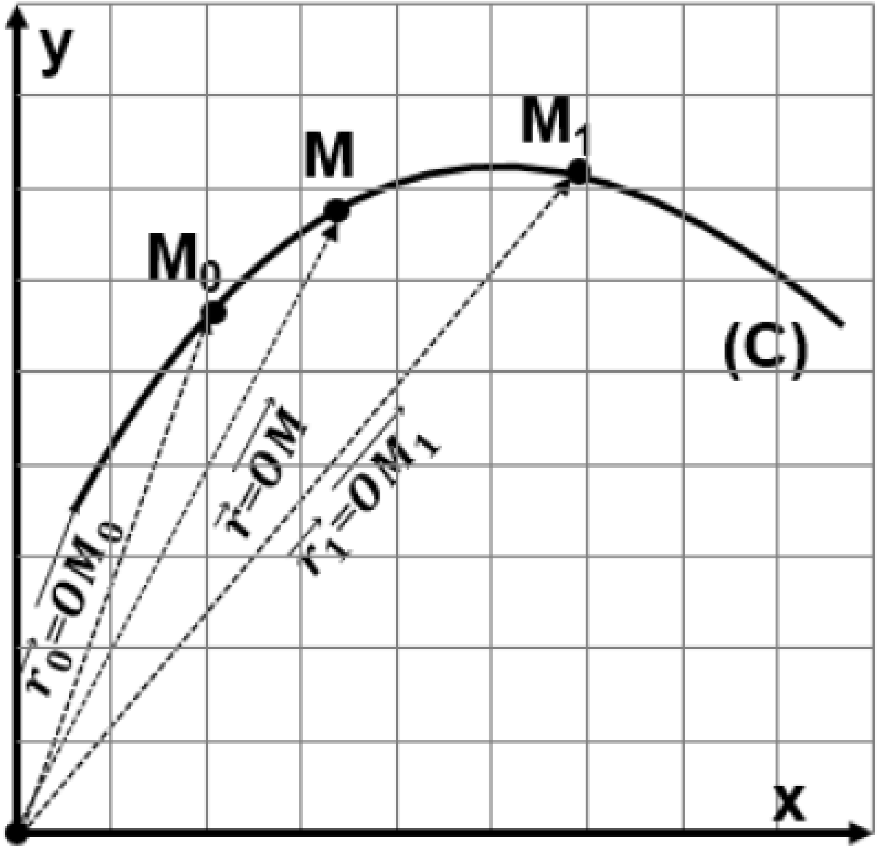
The position of a point M is the physical magnitude that represents where a body is located.

The position vector $\vec{r}$ is defined by:

$$\vec{r} = \overrightarrow{OM} = x.\vec{i} + y.\vec{j}$$

The parametric equations of M gives x and y in terms of time:

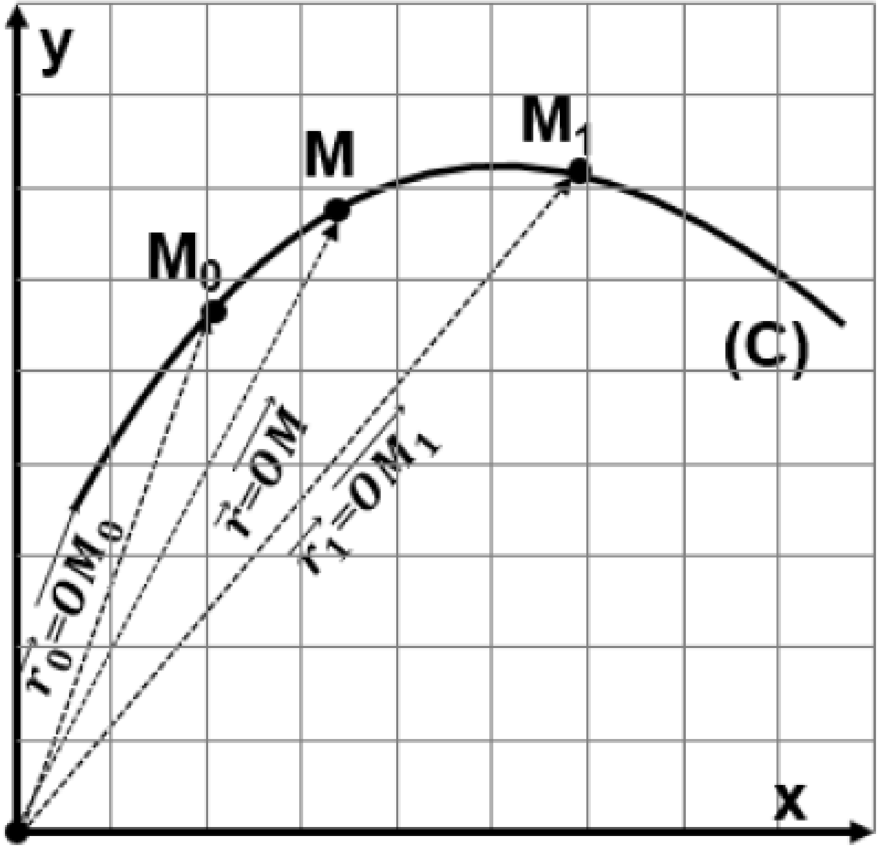
$$M\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$



The position

The magnitude of the position vector represent the distance OM:

$$OM = r = \sqrt{x^2 + y^2}$$



The position

Application 4:

The position vector of a moving point M is:

$$\vec{r} = (t - 2)\vec{i} + (2t + 1)\vec{j}$$

1) Determine the Parametric equations of M:

$x = t - 2$ **And** $y = 2t + 1$

The position

$x = t - 2$ And $y = 2t + 1$

2)Determine the magnitude of the position vector at t=1s.

$OM = r = \sqrt{x^2 + y^2}$ ⇒ $OM = r = \sqrt{(t-2)^2 + (2t+1)^2}$

$OM = r = \sqrt{(1-2)^2 + (2(1)+1)^2}$.

$OM = r = \sqrt{(-1)^2 + (3)^2}$

$OM = r = \sqrt{1+9}$ ⇒ $OM = r = \sqrt{10}cm$

The position

Application 5:

The position vector of a moving point M is:

$$\vec{r} = 2\cos(3t)\vec{i} + 2\sin(3t)\vec{j}$$

1) Determine the Parametric equations of M:

$$x = 2\cos(3t) \qquad y = 2\sin(3t)$$

2) Determine the magnitude of the position vector at $t = \pi$.

$$OM = r = \sqrt{x^2 + y^2} \Rightarrow r = \sqrt{[2cos(3t)]^2 + [2\sin(3t)]^2}.$$

$$OM = r = \sqrt{4cos^2(3\pi) + 4sin^2(3\pi)}$$

The position

$$OM = r = \sqrt{4[cos^2(3\pi) + sin^2(3\pi)]}$$

Math rule:

$$cos^2(\alpha) + sin^2(\alpha) = 1$$

$$OM = r = \sqrt{4[1]}$$

$$OM = r = \sqrt{4}$$

$$OM = r = 2cm$$

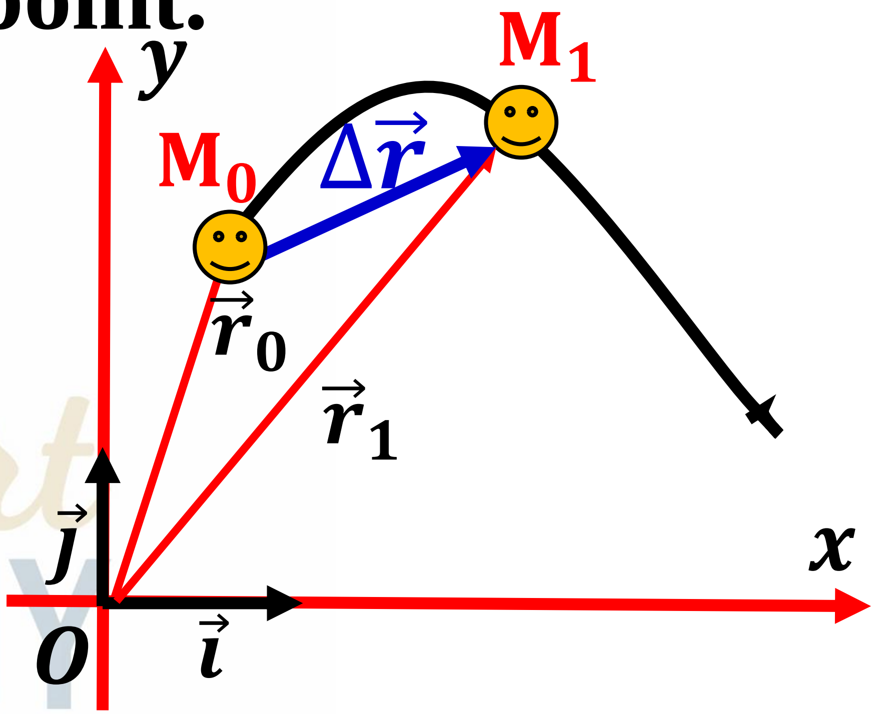
The displacement vector

The displacement vector is the vector joining the initial (M_i) and the final position (M_f) of a moving point.

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta\vec{r} = \overrightarrow{M_iM_f} = \overrightarrow{OM}_f - \overrightarrow{OM}_i$$

$$\Delta\vec{r} = \overrightarrow{M_1M_2} = \overrightarrow{OM}_1 - \overrightarrow{OM}_0$$



The displacement vector

Application 8:

A particle (M) is lunched in the vertical plane (O, $\vec{i}$, $\vec{j}$).

The particle (M) has the position time equation:

$$\vec{r} = \overrightarrow{OM} = (4t + 1)\vec{i} + (2t + 1)\vec{j}$$

1)Determine the expression of the position vectors of M at t_0 and at $t_1 = 1s$.

2)Deduce the displacement vector of M between the two instants.

3)Determine the magnitude of displacement vector.

The displacement vector

$$\vec{r} = \overrightarrow{OM} = (4t+1)\vec{i} + (2t+1)\vec{j}$$

1)Determine the expression of the position vectors of M at t_0 and at $t_1 = 1s$.

$$\vec{r}_0 = \overrightarrow{OM}_0 = [(4 \times 0) + 1]\vec{i} + [(2 \times 0) + 1]\vec{j}$$

$$\vec{r}_0 = \overrightarrow{OM}_0 = 1.\vec{i} + 1.\vec{j}$$

$$\vec{r}_1 = \overrightarrow{OM}_1 = [(4 \times 1) + 1]\vec{i} + [(2 \times 1) + 1]\vec{j}$$

$$\vec{r}_1 = \overrightarrow{OM}_1 = 5.\vec{i} + 3.\vec{j}$$

The displacement vector

$$\vec{r} = \overrightarrow{OM} = (4t+1)\vec{i} + (2t+1)\vec{j}$$

$$\vec{r}_0 = \overrightarrow{OM_0} = 1.\vec{i} + 1.\vec{j}$$

$$\vec{r}_1 = \overrightarrow{OM_1} = 5.\vec{i} + 3.\vec{j}$$

2) Deduce the displacement vector of M between the two instants.

$$\Delta\vec{r} = \vec{r}_1 - \vec{r}_0 = \overrightarrow{M_0M_2} = \overrightarrow{OM_1} - \overrightarrow{OM_0}$$

$$\Delta\vec{r} = 5.\vec{i} + 3.\vec{j} - (1.\vec{i} + 1.\vec{j})$$

$$\Delta\vec{r} = 5.\vec{i} + 3.\vec{j} - 1.\vec{i} - 1.\vec{j}$$

$$\Delta\vec{r} = 4.\vec{i} + 2.\vec{j}$$

The displacement vector

3) Determine the magnitude of displacement vector.

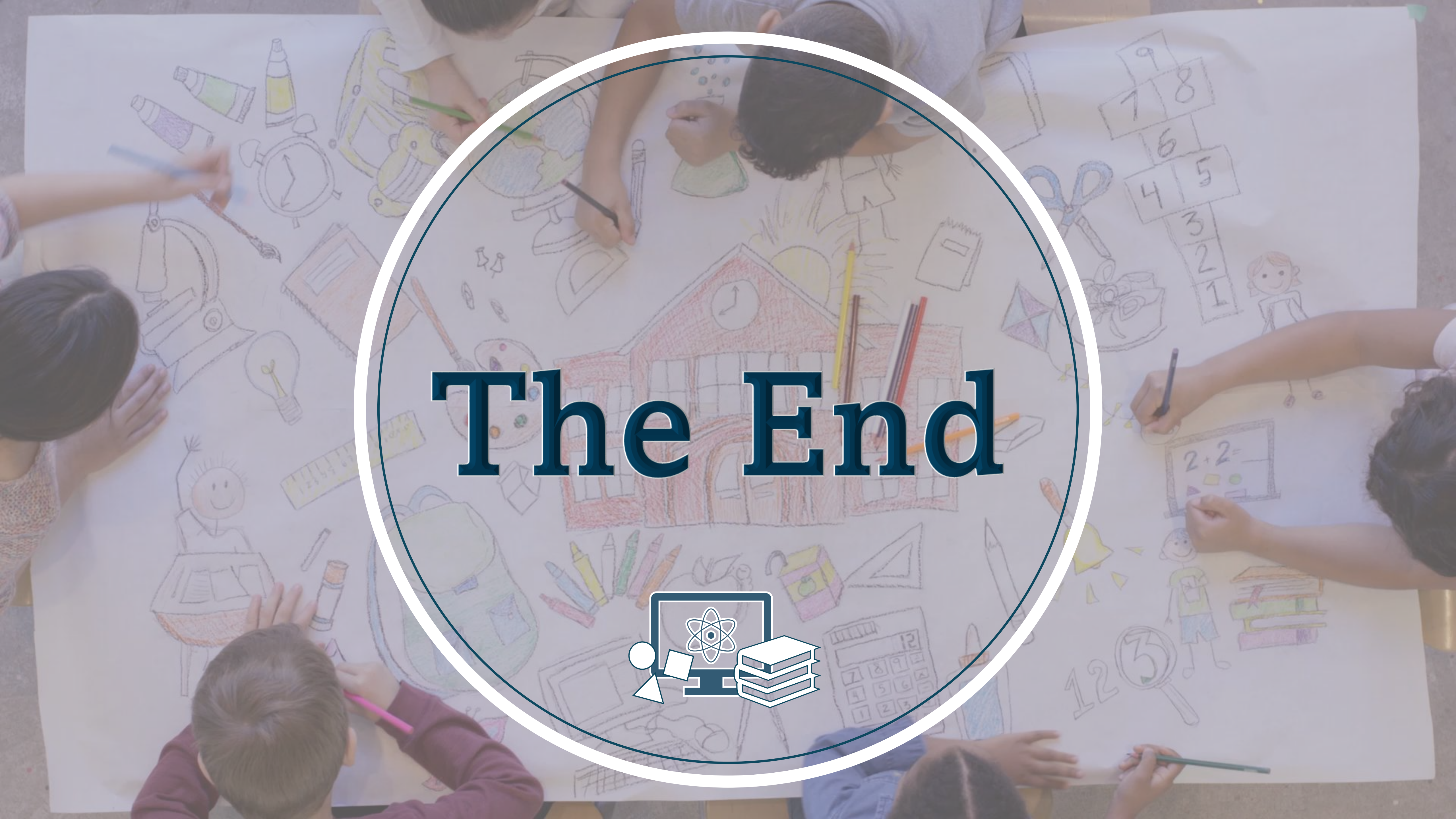
$$\Delta\vec{r} = 4.\vec{i} + 2.\vec{j}$$

$$\Delta r = M_0M_1 = \sqrt{x^2 + y^2}$$

$$\Delta r = M_0M_1 = \sqrt{(4)^2+(2)^2}$$

$$\Delta r = M_0M_1 = \sqrt{16 + 4}$$

$$\Delta r = M_0M_1 = \sqrt{20} \Rightarrow \Delta r = M_0M_1 = 4.5m$$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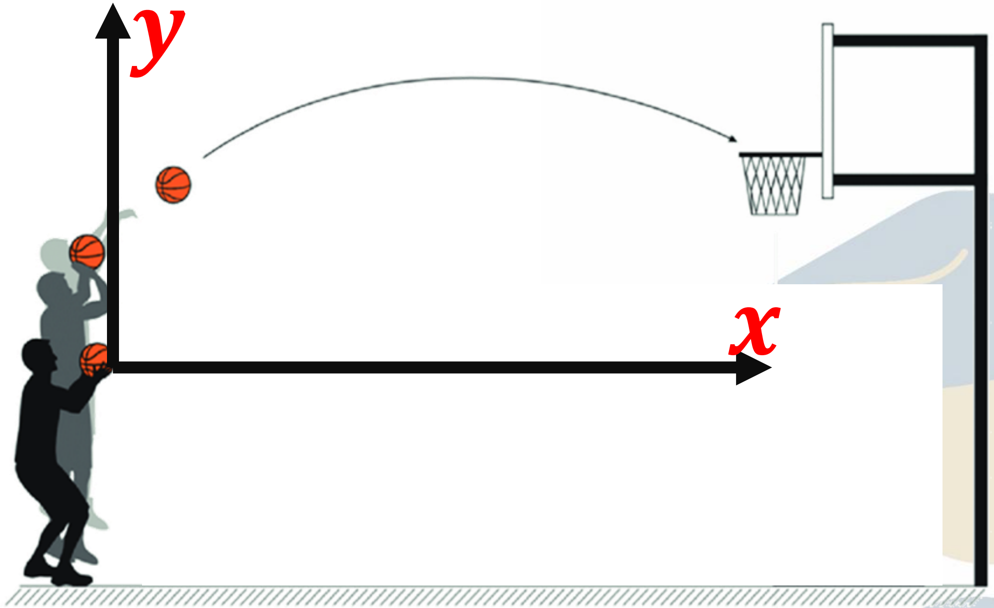


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

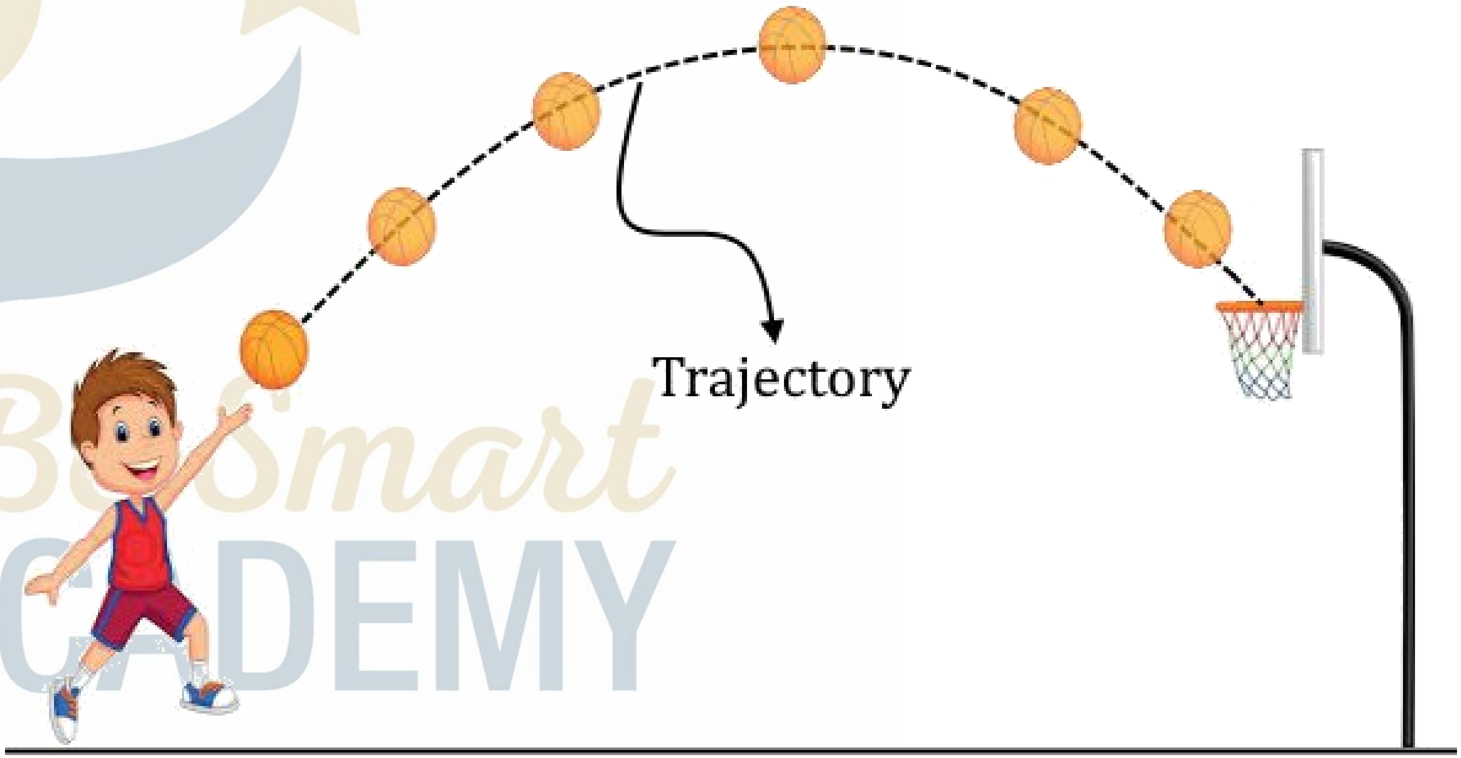
OBJECTIVES

1 Specify the trajectory of motion

The trajectory of motion

Trajectory is the path that the object follows when it is in motion.

The equation of trajectory is a relation between the coordinates (x, y) and independent of time.

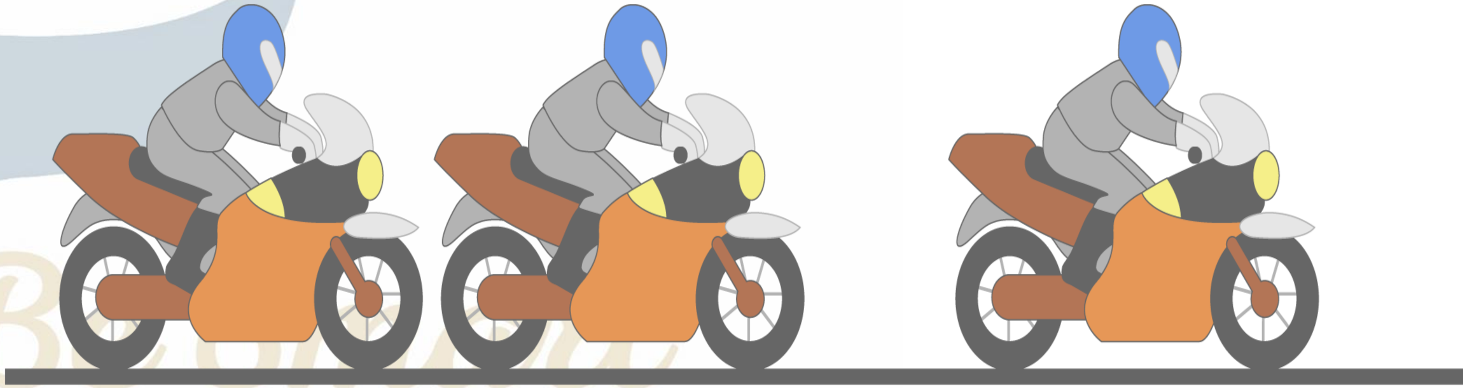


The trajectory of motion

The trajectory may have many shapes as follows:

1. Rectilinear:

When the object moves along a straight line (horizontal, vertical, inclined).



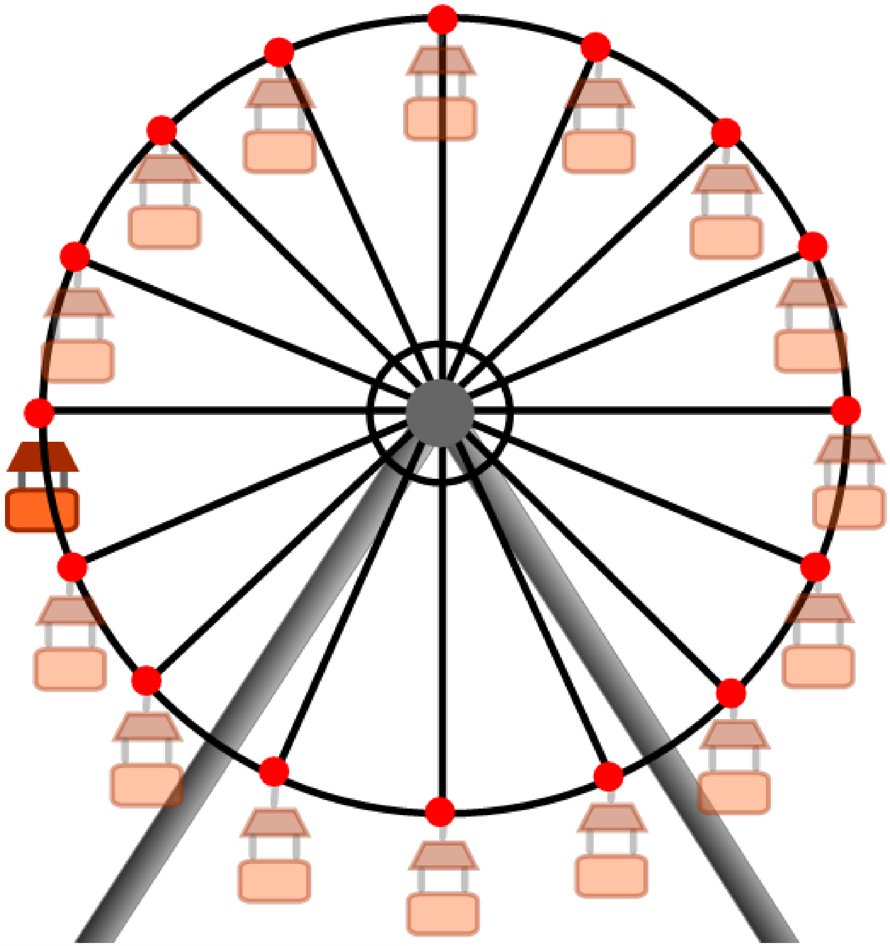
The equation of a line in general is :

$$y = ax + b$$

The trajectory of motion

2.Circular:

When the object assimilated to a particle moves along a circle



The equation of a circle is :

$$(x-a)^2+(y-b)^2=R^2$$

$$x^2+y^2-2ax-2by+c=0$$

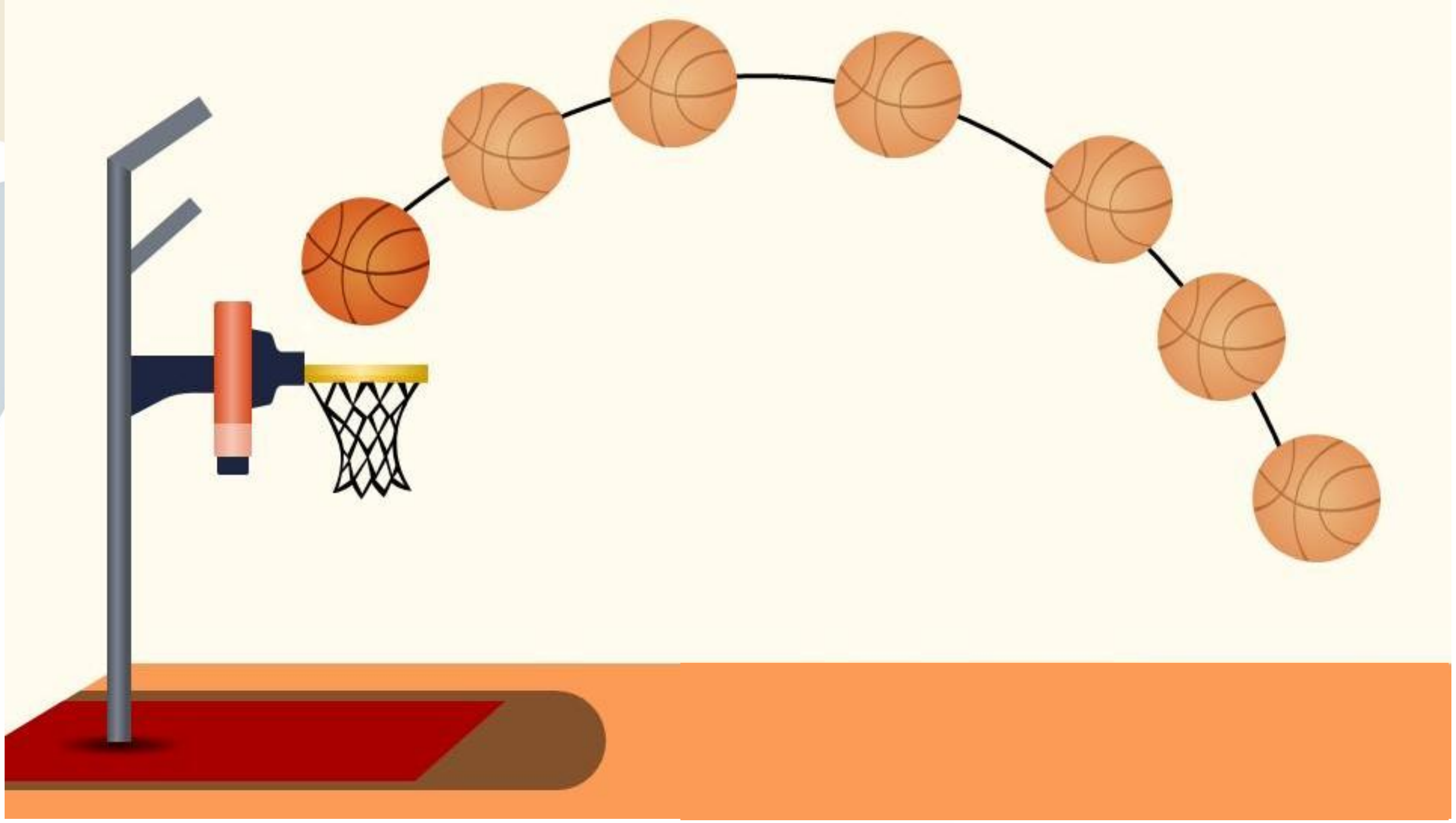
The trajectory of motion

3. Curvilinear (parabola):

When the object assimilated to a particle moves along a curve

The equation of parabola is of the form:

$$y = ax^2 + bx + c$$



The trajectory of motion

Application 6:

The position vector of a moving point M is:

$$\vec{r} = (t - 2)\vec{i} + (2t + 1)\vec{j}$$

Determine the equation of the trajectory of the point M.

$x = t - 2$	And	$y = 2t + 1$

$x = t - 2 \Rightarrow x + 2 = t$

Substitute in y: $y = 2t + 1$

$$y = 2(x + 2) + 1$$

$y = 2x + 4 + 1 \Rightarrow y = 2x + 5$

$$y = ax + b$$

The trajectory is St. line or rectilinear

The trajectory of motion

Application 7:

The position vector of a moving point M is:

$$\vec{r} = 2cos(3t)\vec{i} + 2sin(3t)\vec{j}$$

Determine the equation of the trajectory of the point M.

$x = 2cos(3t)$	And	$y = 2sin(3t)$

$x^2 = 4cos^2(3t)$ … (1)

$y^2 = 4sin^2(3t)$ … (2)

The trajectory of motion

$$x^2 = 4cos^2(3t) \dots (1) \qquad y^2 = 4sin^2(3t) \dots (2)$$

Add the two equations: $x^2 + y^2 = 4cos^2(3t) + 4sin^2(3t)$

$$x^2 + y^2 = 4\left[cos^2(3t) + sin^2(3t)\right]$$

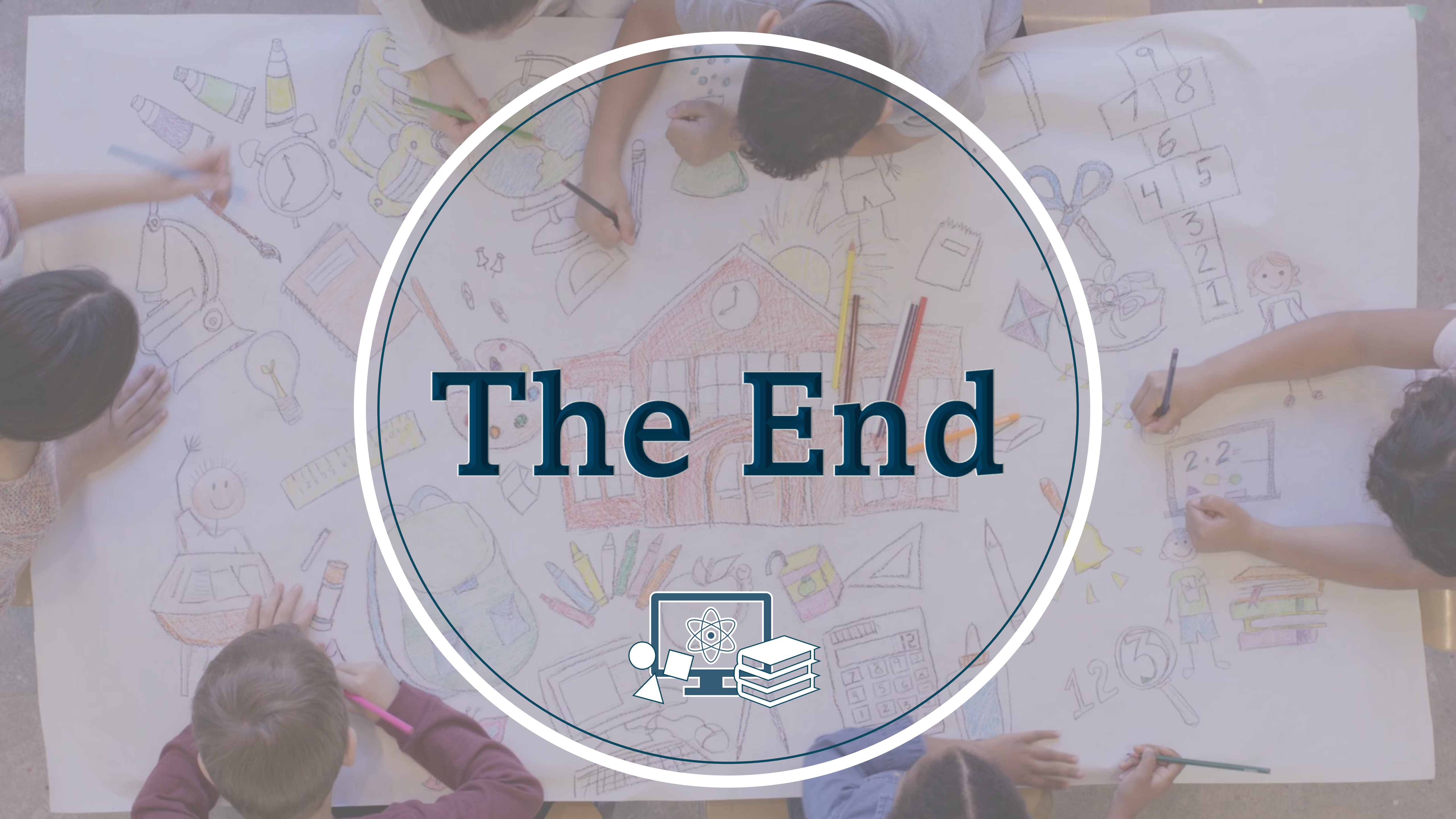
$$x^2 + y^2 = 4[1]$$

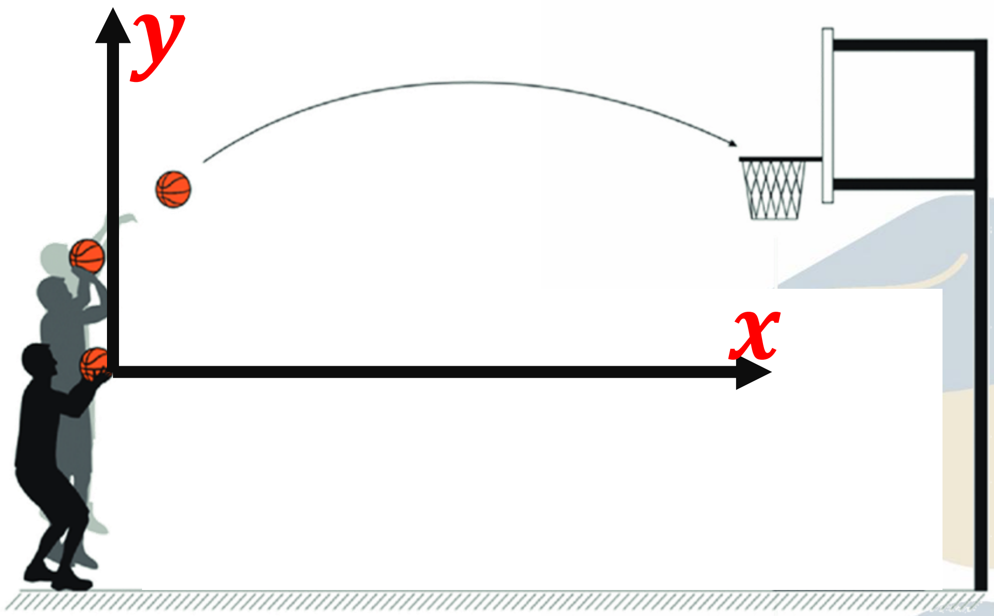
$$x^2 + y^2 = 4$$

$$x^2 + y^2 = 2^2$$

The trajectory is circle of center (0;0) and R=2m

The End





Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

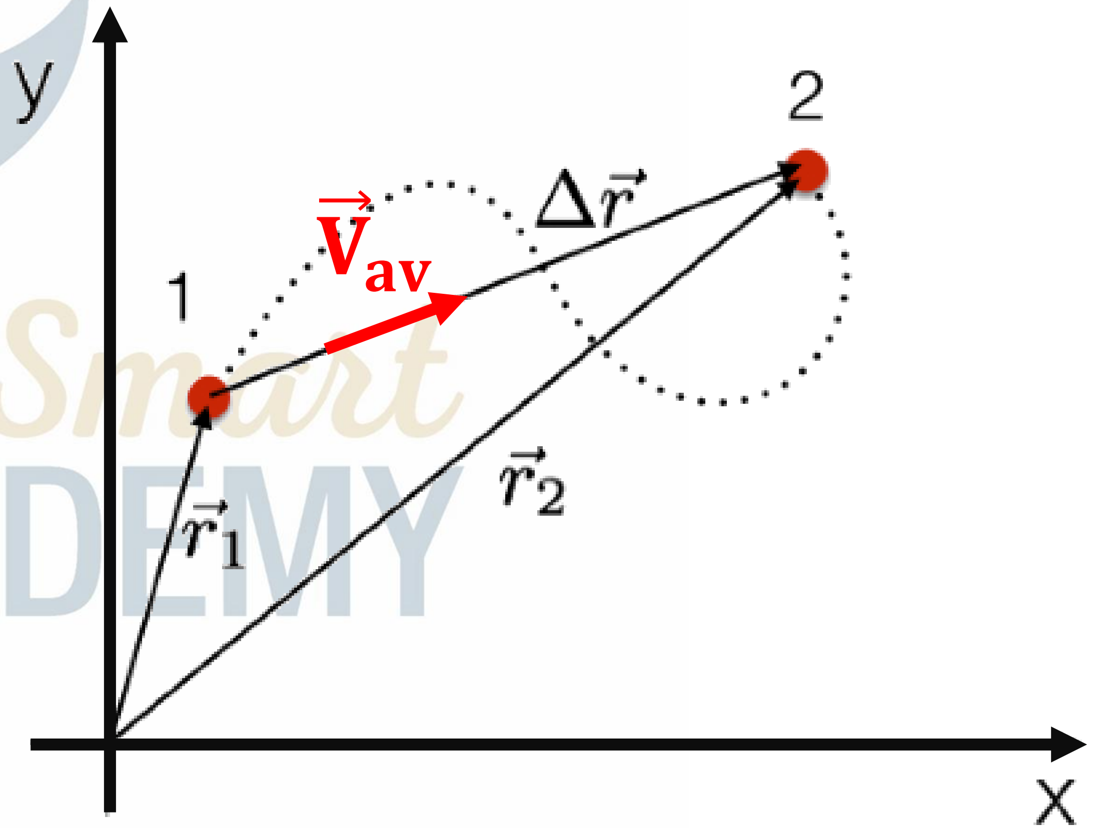
1. Determine average velocity vector and its magnitude
2. Determine the instantaneous velocity vector and its characteristics

The average velocity vector

The average velocity vector is the variation of the position vector during an interval of time.

The average velocity vector is independent from the path.

$\vec{V}_{av}$ and $\overrightarrow{\Delta r}$ are collinear

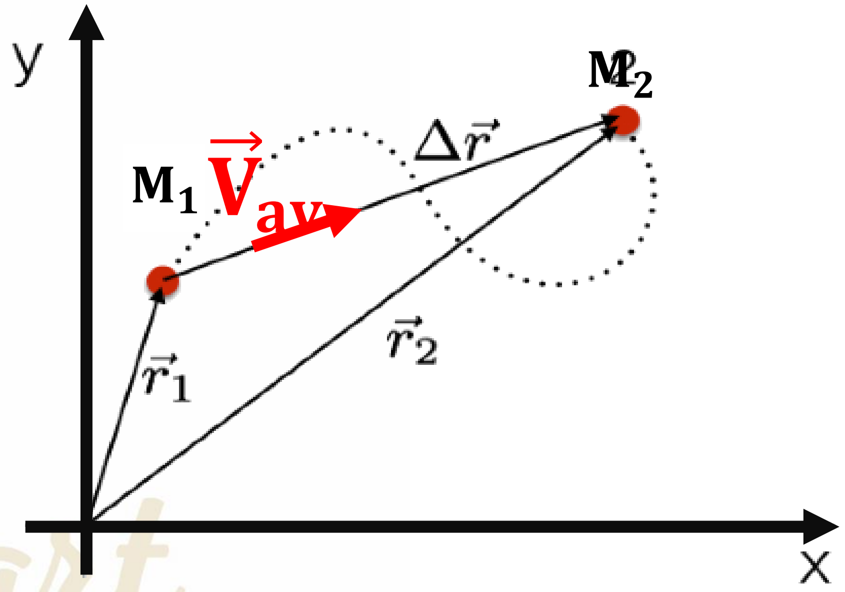


The average velocity vector

The general form of average velocity vector:

$$\vec{V}_{av} = \frac{\overrightarrow{M_iM_f}}{\Delta t} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

$$\vec{V}_{av} = V_x\vec{i} + V_y\vec{j}$$



The value (magnitude) of the average velocity vector is given by :

$$V_{av} = \sqrt{V_x^2 + V_y^2} \quad (\text{ m/s })$$

The average velocity vector

Application 9: Consider the position vector of a point M

$$\overrightarrow{OM}(t) = \vec{r}(t) = (t+1)\vec{i} + (-t^2 + 2t)\vec{j}$$

1)Determine the average velocity vector between t_0 and t_3.

$$\vec{r}_0 = (0+1)\vec{i} + (-(0)^2 + 2(0))\vec{j} \Rightarrow \vec{r}_0 = \vec{i}$$

$$\vec{r}_3 = (3+1)\vec{i} + (-(3)^2 + 2(3))\vec{j} \Rightarrow \vec{r}_3 = 4\vec{i} - 3\vec{j}$$

$$\vec{V}_{0\to 3} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\vec{r}_3 - \vec{r}_0}{t_3 - t_0} = \frac{(4\vec{i} - 3\vec{j}) - 1\vec{i}}{3-0} = \frac{3\vec{i} - 3\vec{j}}{3}$$

$$\vec{V}_{av(0\to 3)} = \vec{i} - \vec{j}$$

The average velocity vector

2) Deduce its value.

$$\vec{V}_{av(0\to 3)} = 1\vec{i} - 1\vec{j}$$

$$V_{av} = \sqrt{V_x^2 + V_y^2} \Rightarrow V_{av} = \sqrt{(1)^2 + (-1)^2}$$

$$V_{av} = \sqrt{1+1}$$

$$V_{av} = \sqrt{2}\ m/s$$

The average velocity vector

Difference between Average Velocity and Average speed

Speed is a scalar quantity and has only magnitude

Velocity, is a vector quantity and so has both magnitude and direction.

Average velocity is different from average speed in that it considers the direction of travel and the overall change in position.

The instantaneous velocity vector

Instantaneous velocity vector is the variation of the position vector during a very small interval of time dt

The velocity vector in general: $\vec{V} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$

For instantaneous velocity vector the interval; of time is very small $\Delta t \to 0$

$$\vec{V}_{in} = lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = lim_{\Delta t \to 0} \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$$

The instantaneous velocity vector

$$\vec{V}_{in} = lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = lim_{\Delta t \to 0} \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Compare this equation to the equation of derivative (by definition) in math :

$$lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x) = \frac{df(x)}{dx}$$

$$\vec{V}_{in} = lim_{\Delta t \to 0} \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \vec{r}'(t) = \frac{d\vec{r}}{dt}$$

The instantaneous velocity vector

$$\vec{V}_{in} = \vec{r}'(t) = \frac{d\vec{r}}{dt}$$

$$\vec{V}(t) = \overrightarrow{r'}(t) = V_x\vec{i} + V_y\vec{j}$$

Characteristics of the instantaneous velocity vector:

- **Point of application: The considered point.**
- **Line of action: <u>tangent</u> to the trajectory at this point.**
- **Direction: along the motion.**
- **Magnitude (value) :**

$$V = \sqrt{V_x^2 + V_y^2}\ (\mathrm{m}/s)$$

The instantaneous velocity vector

Application 10:

Consider the position vector of a point M:

$$\overrightarrow{OM}(t) = \vec{r}(t) = (t+1)\vec{i} + (-t^2 + 2t)\vec{j}$$

1)Determine the velocity vector at any instant t.

The instantaneous velocity vector is the derivative of position vector

$$\vec{V}(t) = \vec{r'}(t)$$

$$\vec{V}(t) = 1\vec{i} + (-2t + 2)\vec{j}$$

The instantaneous velocity vector

$$\vec{V}(t) = 1\vec{i} + (-2t + 2)\vec{j}$$

2)Determine V_1, and V_2

We must find the vectors $\vec{V}_1$ and $\vec{V}_2$ then their values

$$\vec{V}_1 = 1\vec{i} + (-2(1) + 2)\vec{j} \Rightarrow \vec{V}_1 = 1\vec{i}$$

$$V_1 = \sqrt{V_x^2 + V_y^2} = \sqrt{1^2 + 0^2}$$

$$V_1 = 1m/s$$

The instantaneous velocity vector

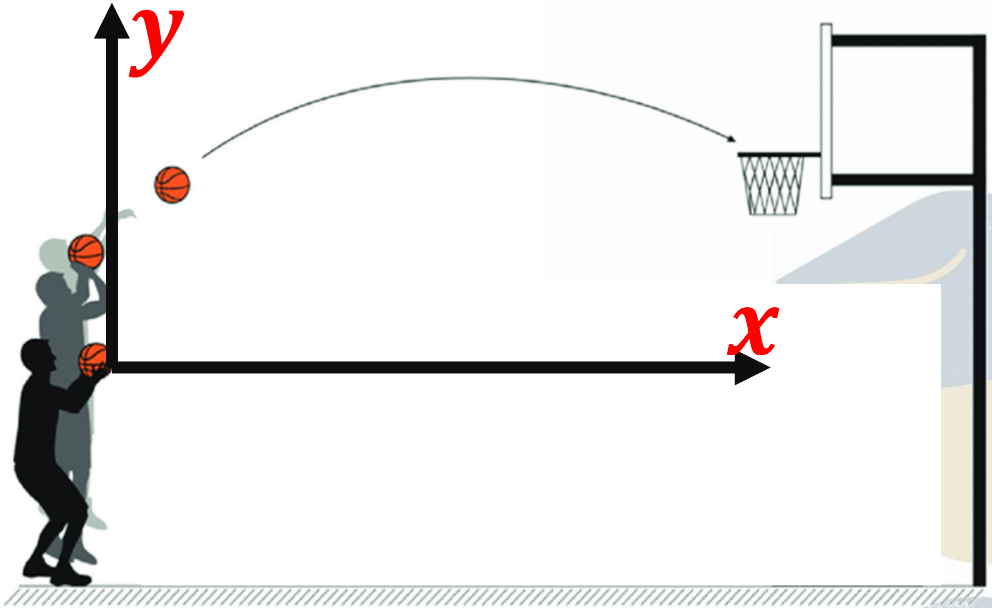
$$\vec{V}(t) = 1\vec{i} + (-2t + 2)\vec{j}$$

$$\vec{V}_2 = 1\vec{i} + (-2(2) + 2)\vec{j} \Rightarrow \vec{V}_2 = 1\vec{i} - 2\vec{j}$$

$$V_2 = \sqrt{V_x^2 + V_y^2} = \sqrt{1^2 + (-2)^2}$$

$$V_2 = \sqrt{1 + 4}$$

$$V_2 = \sqrt{5} m/s$$



Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Exercise 1:

Given two vectors $\vec{F}_1 = 15\vec{i} + 10\vec{j}$ and $\vec{F}_2 = 16\vec{i} - 8\vec{j}$.

1. Determine the vector $\vec{F}$, such that $\vec{F} = \vec{F}_1 + 3\,\vec{F}_2$.
2. Calculate the magnitude of the vector $\vec{F}$.
3. Determine the angle α between the vector $\vec{F}$ and the x-axis.
4. Determine the derivative of the following functions.

a) $f(t) = -3t^2 + 2t - 1$

b) $f(t) = \sqrt{5t^3 - 4t + 4}$

c) $f(t) = 3\cos(3t^2 + 2t)$

$\vec{F}_1 = 15\vec{i} + 10\vec{j}$ and $\vec{F}_2 = 16\vec{i} - 8\vec{j}$.

1. Determine the vector $\vec{F}$, such that $\vec{F} = \vec{F}_1 + 3\,\vec{F}_2$.

$$\vec{F} = \vec{F}_1 + 3\,\vec{F}_2 = 15\vec{i} + 10\vec{j} + 3(16\vec{i} - 8\vec{j})$$

$$\vec{F} = 15\vec{i} + 10\vec{j} + 48\vec{i} - 24\vec{j}$$

$$\vec{F} = 63\vec{i} - 14\vec{j}$$

2. Calculate the magnitude of the vector $\vec{F}$.

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(63)^2 + (-14)^2}$$

$$F = \sqrt{3969 + 196} = \sqrt{4165}$$

$$F = 64.5N$$

$\vec{F}_1 = 15\vec{i} + 10\vec{j}$ and $\vec{F}_2 = 16\vec{i} - 8\vec{j}$; $\vec{F} = 63\vec{i} - 14\vec{j}$

3. Determine the angle α between the vector $\vec{F}$ and the x-axis.

$$\tan(\alpha) = \frac{F_y}{F_x} = \frac{-14}{63} \Rightarrow \tan(\alpha) = -0.222$$

$$\alpha = -12.5^\circ$$

4. Determine the derivative of the following functions.

a) $f(t) = -3t^2 + 2t - 1$

$$f(t) = -3t^2 + 2t - 1 \rightarrow f'(t) = -3(2t) + 2(1)$$

$$f'(t) = -6t + 2$$

a) $f(t) = \sqrt{5t^3 - 4t + 4}$

Let $u = 5t^3 - 4t + +4$

$$u' = 5(3t^2) - 4(1) + 0$$

$$u' = 15t^2 - 4$$

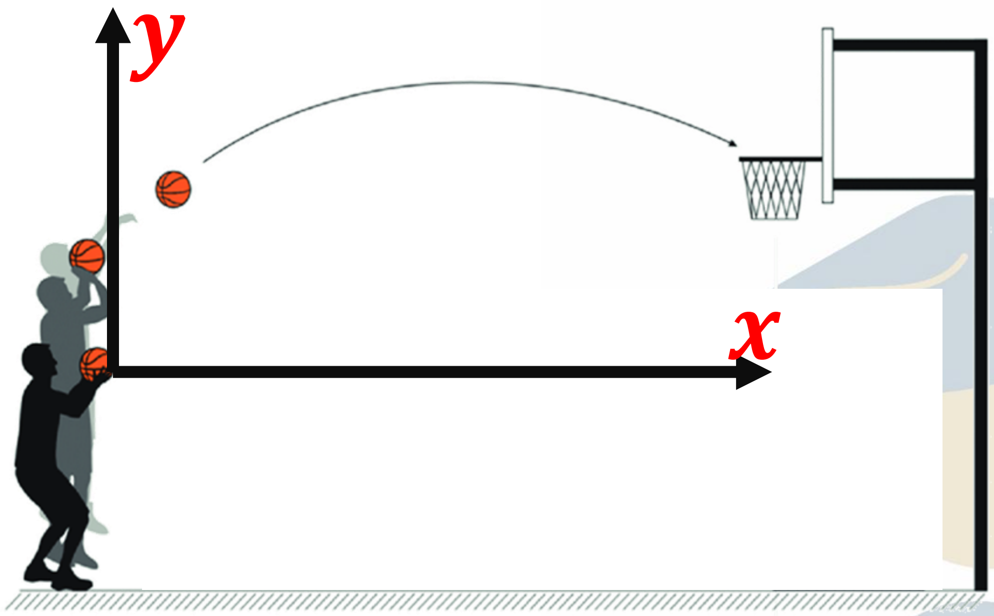
$$f'(t) = \frac{u'}{2\sqrt{u}} = \frac{15t^2 - 4}{2\sqrt{5t^3 - 4t + 4}}$$

b) $f(t) = 3\cos(3t^2 + 2t)$

$$f'(t) = -3[3(2t) + 2(1)]\sin(3t^2 + 2t)$$

$$f'(t) = -3[6t + 2]\sin(3t^2 + 2t)$$

$$f'(t) = (-18t - 6)\sin(3t^2 + 2t)$$



Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Exercise 2:

The aim of this exercise is to determine the radius of curvature of a trajectory during the motion of a particle M.

The position vector of a moving particle M, in a reference (O, $\vec{i}, \vec{j}$), is given by:

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + \left(-4t^2 + 2t\right).\vec{j} \qquad \textit{in SI unit}$$

1) Determine the position vector of the particle at the instants $t_0 = 0s$ and $t_2 = 2s$.

2) Determine the average velocity vector between the two instants.

3) Determine the equation of trajectory of the particle (M). Deduce its shape.

4) Determine the velocity vector at the instant t, then the value of the speed in terms of t.

5) Derive the acceleration vector of the particle M. deduce its magnitude.

$$\vec{r} = 2t.\vec{i} + (-4t^2 + 2t).\vec{j}$$

1) Determine the position vector of the particle at the instants $t_0 = 0s$ and $t_2 = 2s$.

At $t_0 = 0$

$$\vec{r}_0 = 2(0).\vec{i} + (-4(0)^2 + 2(0)).\vec{j}$$

$$\vec{r}_0 = 0.\vec{i} + 0.\vec{j}$$

At $t_2 = 2$

$$\vec{r}_2 = 2(2).\vec{i} + (-4(2)^2 + 2(2)).\vec{j}$$

$$\vec{r}_2 = 4.\vec{i} + (-4(4) + 4).\vec{j}$$

$$\vec{r}_2 = 4.\vec{i} + (-16 + 4).\vec{j}$$

$$\vec{r}_2 = 4.\vec{i} - 12.\vec{j}$$

$\vec{r} = 2t.\vec{i} + (-4t^2 + 2t).\vec{j}$; $\vec{r}_0 = 0.\vec{i} + 0.\vec{j}$; $\vec{r}_2 = 4.\vec{i} - 12.\vec{j}$

2) Determine the average velocity vector between the two instants

$$\vec{V}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_0}{t_2 - t_0}$$

$$\vec{V}_{av} = \frac{4.\vec{i} - 12.\vec{j} - (0.\vec{i} + 0.\vec{j})}{2 - 0}$$

$$\vec{V}_{av} = \frac{4.\vec{i} - 12.\vec{j}}{2}$$

$$\vec{V}_{av} = 2.\vec{i} - 6.\vec{j}$$

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + (-4t^2 + 2t).\vec{j}$$

3) Determine the equation of trajectory of the particle (M). Deduce its shape.

$x = 2t \rightarrow t = \frac{x}{2}$

Substitute $t = \frac{x}{2}$ in y

$$y = -4t^2 + 2t \rightarrow y = -4\left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)$$

$$y = -4\left(\frac{x^2}{4}\right) + 2\left(\frac{x}{2}\right)$$

$$y = -x^2 + x$$

The obtained equation in the form of $y = ax^2 + bx + c$

The trajectory is parabola

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + (-4t^2 + 2t).\vec{j}$$

3) Determine the velocity vector at the instant t, then the value of the speed in terms of t.

The velocity vector is the derivative of position vector.

$$\vec{V} = \vec{r}' = 2(1).\vec{i} + (-4(2t) + 2(1)).\vec{j}$$

$$\vec{V} = \vec{r}' = 2.\vec{i} + (-8t + 2).\vec{j}$$

The speed is the magnitude of velocity vector

$$V = \sqrt{V_x^2 + V_y^2}$$

$$V = \sqrt{(2)^2 + (-8t + 2)^2}$$

$$V = \sqrt{4 + 64t^2 + 4 - 32t}$$

$$V = \sqrt{64t^2 - 32t + 8}\ \text{m/s}$$

$$\vec{V} = \vec{r}' = 2.\vec{i} + (-8t + 2).\vec{j}$$

4) Derive the acceleration vector of the particle M. deduce its magnitude.

The acceleration vector is the derivative of velocity vector.

$$\vec{a} = \vec{V}' = (0).\vec{i} + (-8(1) + (0)).\vec{j}$$

$$\vec{a} = \vec{V}' = 0.\vec{i} - 8.\vec{j}$$

$$a = \sqrt{a_x^2 + a_y^2}$$

$$a = \sqrt{(0)^2 + (-8)^2}$$

$$a = \sqrt{+64}$$

$$a = \sqrt{64}$$

$$a = 8m / s^2$$

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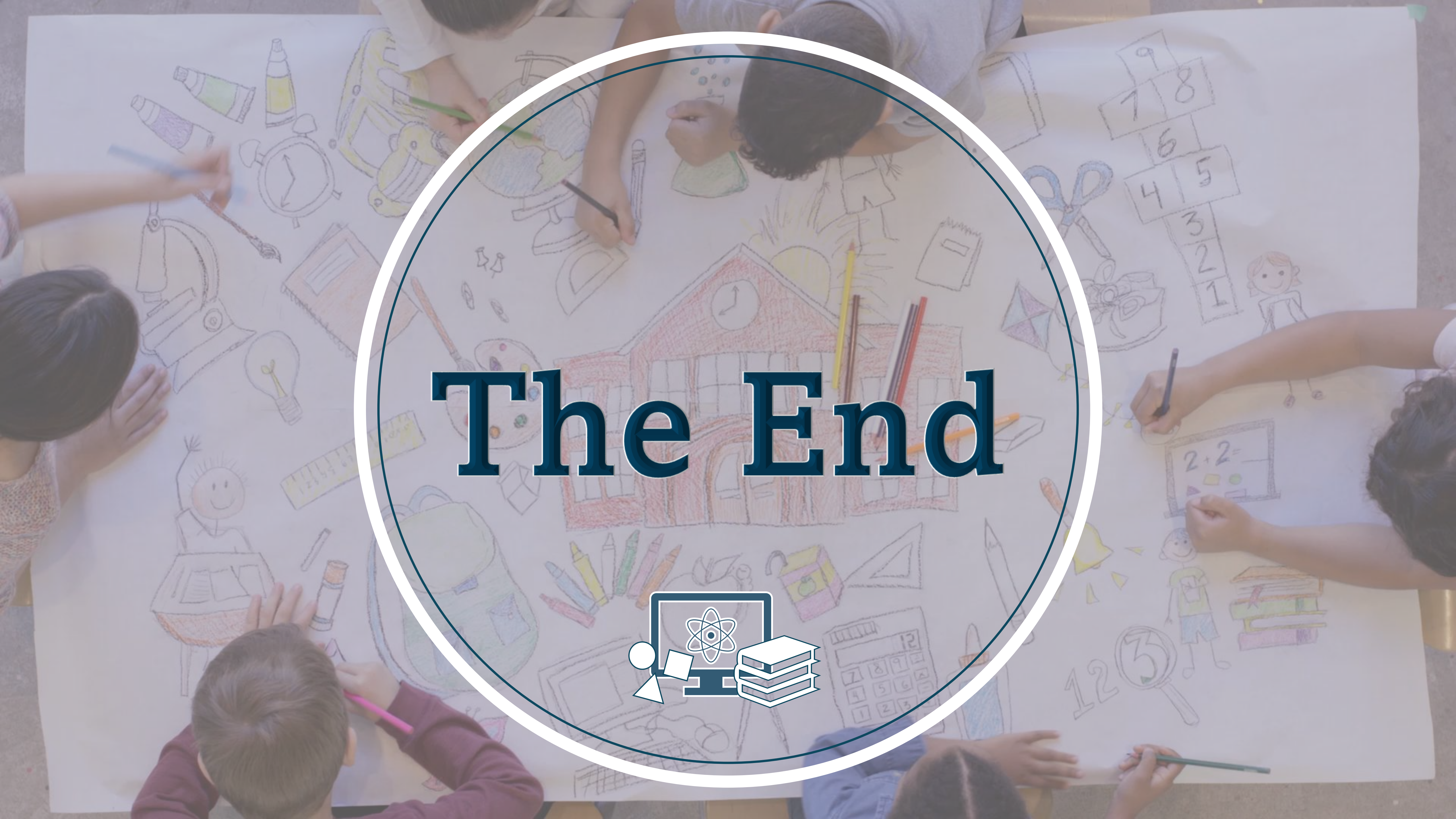
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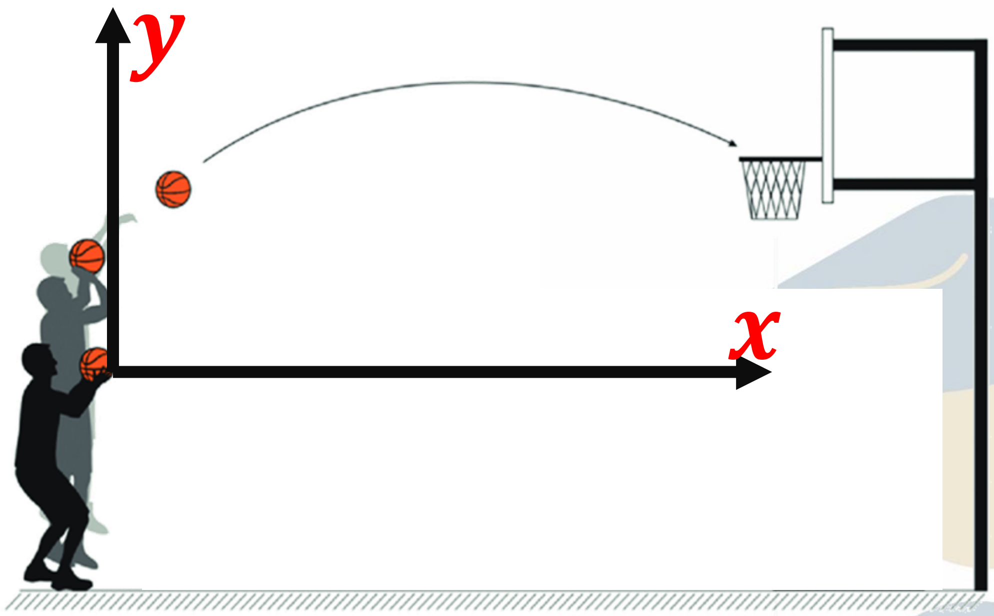


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

1. Determine average acceleration vector and its magnitude
2. Determine the instantaneous acceleration vector and its characteristics

The average acceleration vector

Acceleration vector: It is the variation of the velocity vector during a certain interval of time.

$$\vec{a} = \frac{variation\ of\ velocity\ vector}{interval\ of\ time} = \frac{\Delta\vec{V}}{\Delta t}$$

For a big interval of time Δt:
We talk about average acceleration vector

For small interval of time dt:
We talk about instantaneous acceleration vector

The average acceleration vector

Average acceleration vector is the variation of the velocity vector during a big interval of time. It is given by :

$$\vec{a}_{av} = \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i} = a_x\vec{i} + a_y\vec{j}$$

The average acceleration vector has the same line of action and direction as $\overrightarrow{\Delta V}$

The value (magnitude) of the average acceleration vector is:

$$a_{av} = \sqrt{a_x^2 + a_y^2}\ (m/s^2)$$

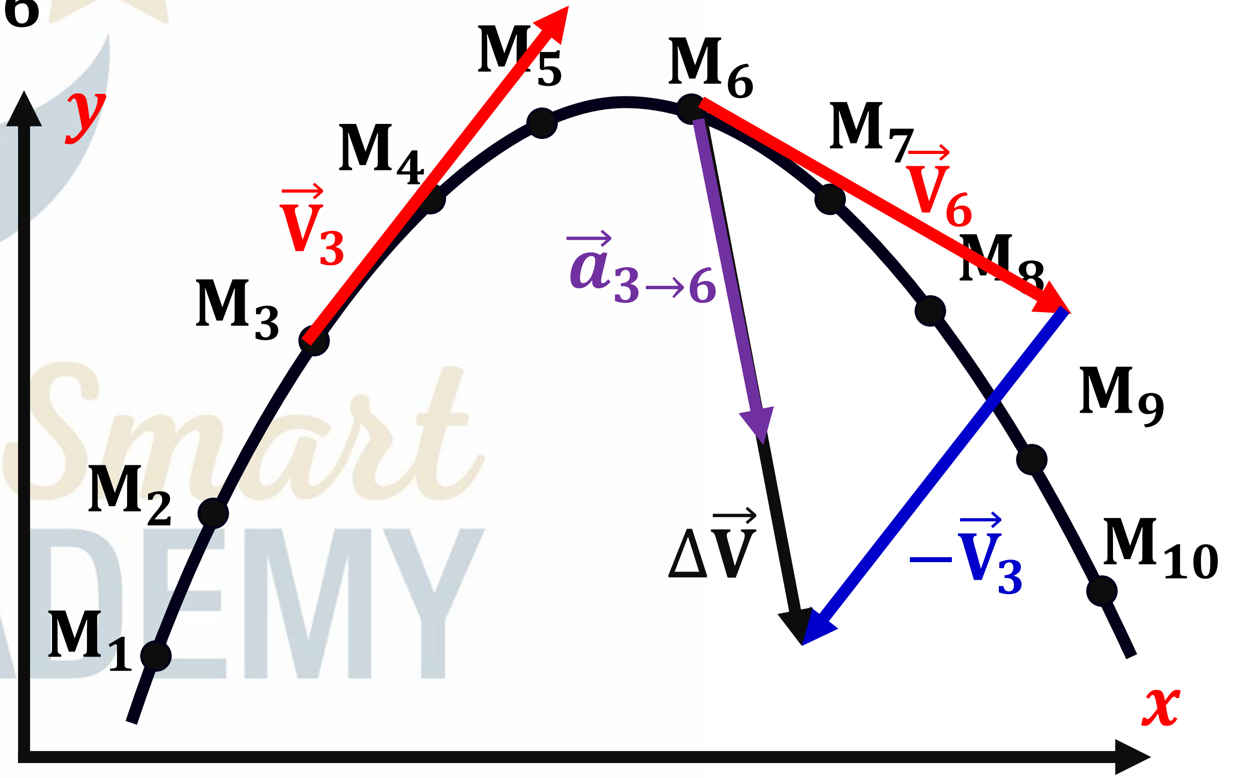
The average acceleration vector

Representation of the vector: $\vec{a}_{av(3\to6)} = \dfrac{\overrightarrow{\Delta V}}{\Delta t} = \dfrac{\vec{V}_6 - \vec{V}_3}{t_6 - t_3}$

1. We trace the vectors $\vec{V}_3$ and $\vec{V}_6$
2. We trace $-\vec{V}_3$
3. Complete the triangle to trace the sum

$$\overrightarrow{\Delta V} = \vec{V}_6 + (-\vec{V}_3)$$

4. The average acceleration vector has same line of action and direction as $\overrightarrow{\Delta V}$



The average acceleration vector

Application 11: Consider a position vector of a moving point: $\vec{r}(t) = (t+1)\vec{i} + (-t^2 + 2t)\vec{j}$

1) Determine the instantaneous velocity vector at instant t.

The instantaneous velocity vector is derivative of position vector:

$$\vec{V} = \vec{r}' = 1\vec{i} + (-2t+2)\vec{j}$$

2) Calculate the value of the average acceleration between t_1 and $t_4 = 4s$

$$\vec{V}_1 = 1\vec{i} + (-2(1)+2)\vec{j} \Rightarrow \vec{V}_1 = \vec{i} + (-2+2)\vec{j}$$

$$\vec{V}_1 = 1\vec{i}$$

The average acceleration vector

$$\vec{V}(t) = 1\vec{i} + (-2t + 2)\vec{j}$$

$$\vec{V}_4 = 1\vec{i} + (-2(4) + 2)\vec{j} \Rightarrow \vec{V}_4 = \vec{i} + (-8 + 2)\vec{j}$$

$$\vec{V}_4 = 1\vec{i} - 6\vec{j}$$

$$\vec{a}_{ave(1\to4)} = \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{\vec{V}_4 - \vec{V}_1}{t_4 - t_1} \Rightarrow \vec{a}_{ave(1\to4)} = \frac{\vec{i} - 6\vec{j} - \vec{i}}{4 - 1} = \frac{-6\vec{j}}{3}$$

$$\vec{a}_{ave(1\to4)} = -2\vec{j}$$

$$a_{ave} = \sqrt{a_x^2 + a_y^2} \Rightarrow a_{ave} = \sqrt{0^2 + (-2)^2} = 2\ m/s^2$$

The instantaneous acceleration vector

The acceleration vector is the variation of the velocity vector during a very small interval of time

$$\vec{a} = \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i}$$

For instantaneous acceleration vector, the time interval is very small: $\Delta t \to 0$

$$\vec{a}_{inst} = lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta V}}{\Delta t} = lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta V}}{\Delta t} = lim_{t_f \to t_i} \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i}$$

The instantaneous acceleration vector

$$\vec{a}_{inst} = lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta V}}{\Delta t} = lim_{t_f \to t_i} \frac{\overrightarrow{\Delta V}}{\Delta t} = lim_{t_f \to t_i} \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i}$$

Compare this equation to that of the definition of derivative:

$$lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x) = \frac{df}{dx}$$

$$\vec{a}_{inst}(t) = lim_{t_f \to t_i} \frac{\vec{V}_f - \vec{V}_i}{t_f - t_i} = \overrightarrow{V'}(t) = \frac{d\vec{V}}{dt}$$

The instantaneous acceleration vector

$$\vec{a}(t) = \overrightarrow{V'}(t) = a_x\vec{i} + a_y\vec{j}$$

The Characteristics of acceleration vector :

- Point of application: The considered point
- Line of action and direction: along $\overrightarrow{\Delta V}$
- Magnitude (value):

$$\mathrm{a} = \sqrt{a_x^2 + a_y^2}\,(\mathrm{m}/s^2)$$

The instantaneous acceleration vector

Application 12: consider the position vector of a point M:

$$\vec{r}(t) = (2t + 1)\vec{i} + (t^2 + 2t)\vec{j}$$

1)Determine the acceleration vector at any instant t.

The acceleration vector is the derivative of velocity vector:

Then we should determine the velocity vector first, then the acceleration vector

$$\vec{V} = \vec{r}'(t) = 2\vec{i} + (2t + 2)\vec{j}$$

$$\vec{a}(t) = \vec{V'}(t) = 0\vec{i} + 2\vec{j}$$

$$\vec{a}(t) = 2\vec{j}$$

The instantaneous acceleration vector

2)Calculate the acceleration a_1 and a_2

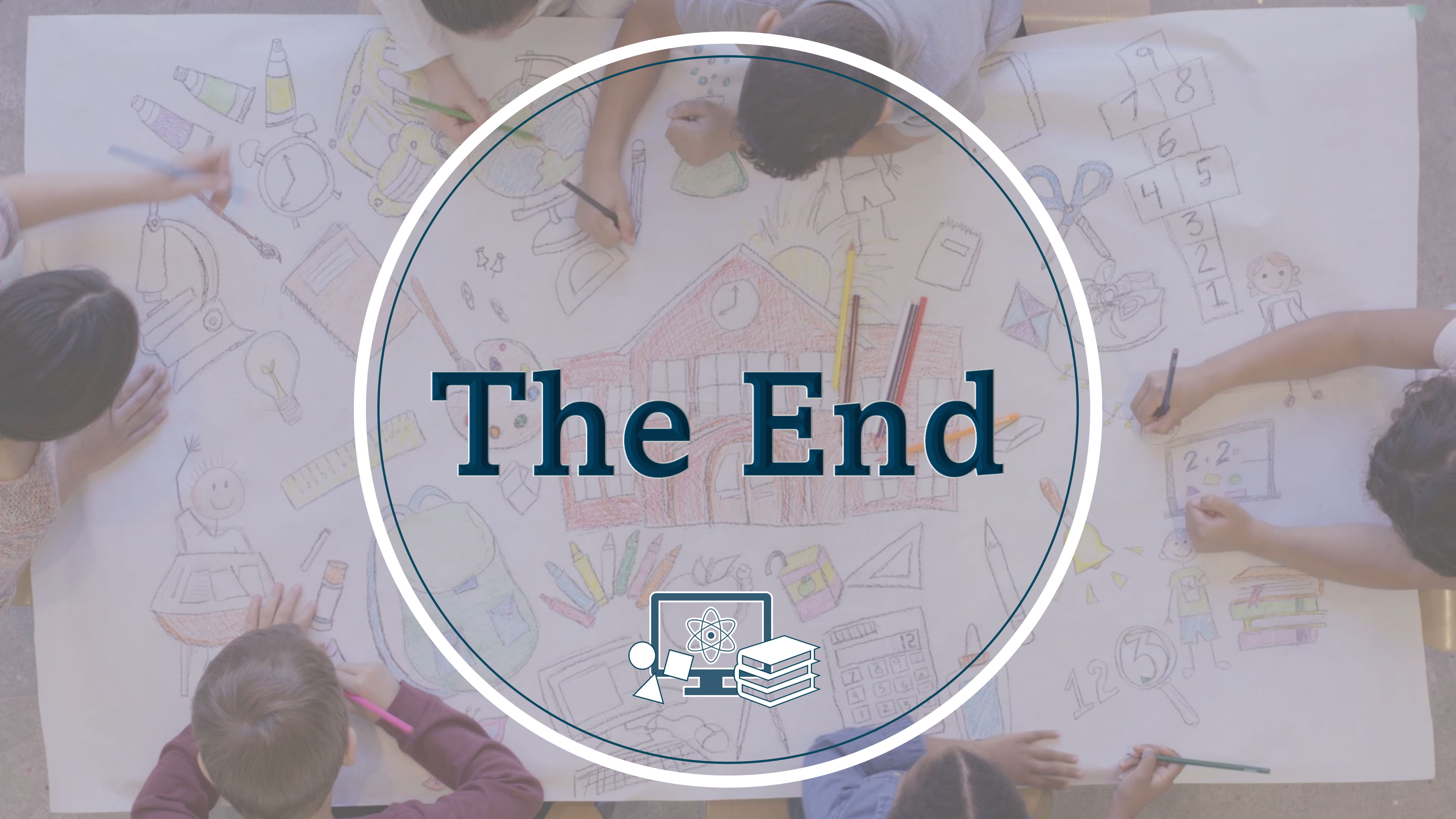
$$\vec{a}(t) = -2\vec{j}$$

We notice that $\vec{a}(t)$ is independant of time then:

$$\vec{a}_1 = -2\vec{j} \qquad \vec{a}_2 = -2\vec{j}$$

$$a_1 = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-2)^2} \Rightarrow a_1 = 2\ m/s^2$$

$$a_2 = \sqrt{a_x^2 + a_y^2} = \sqrt{0^2 + (-2)^2} \Rightarrow a_2 = 2\ m/s^2$$



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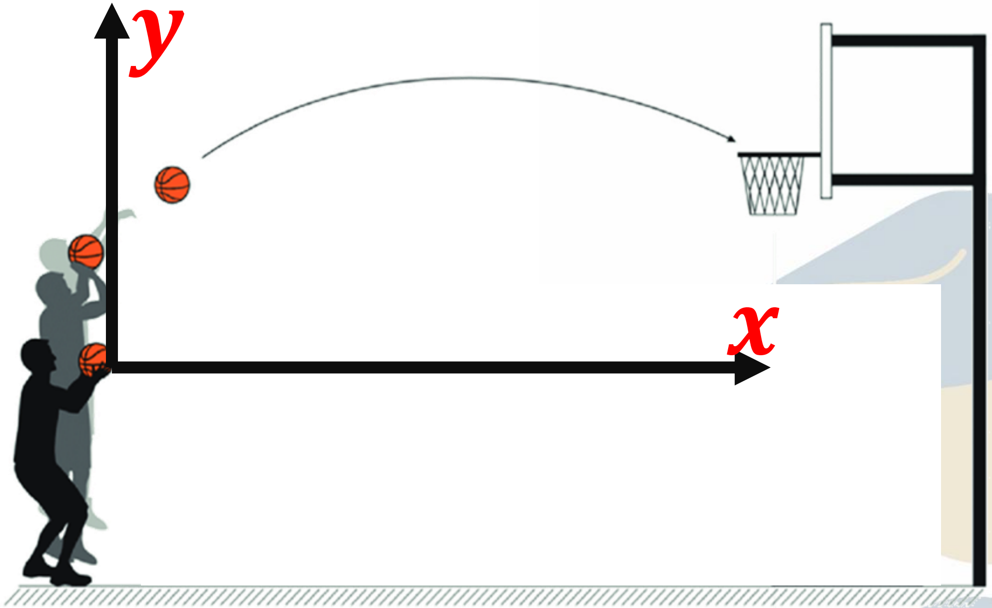


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

1. Determine position and displacement in curvilinear system
2. Determine the speed and velocity vector in curvilinear system

position and displacement in curvilinear system

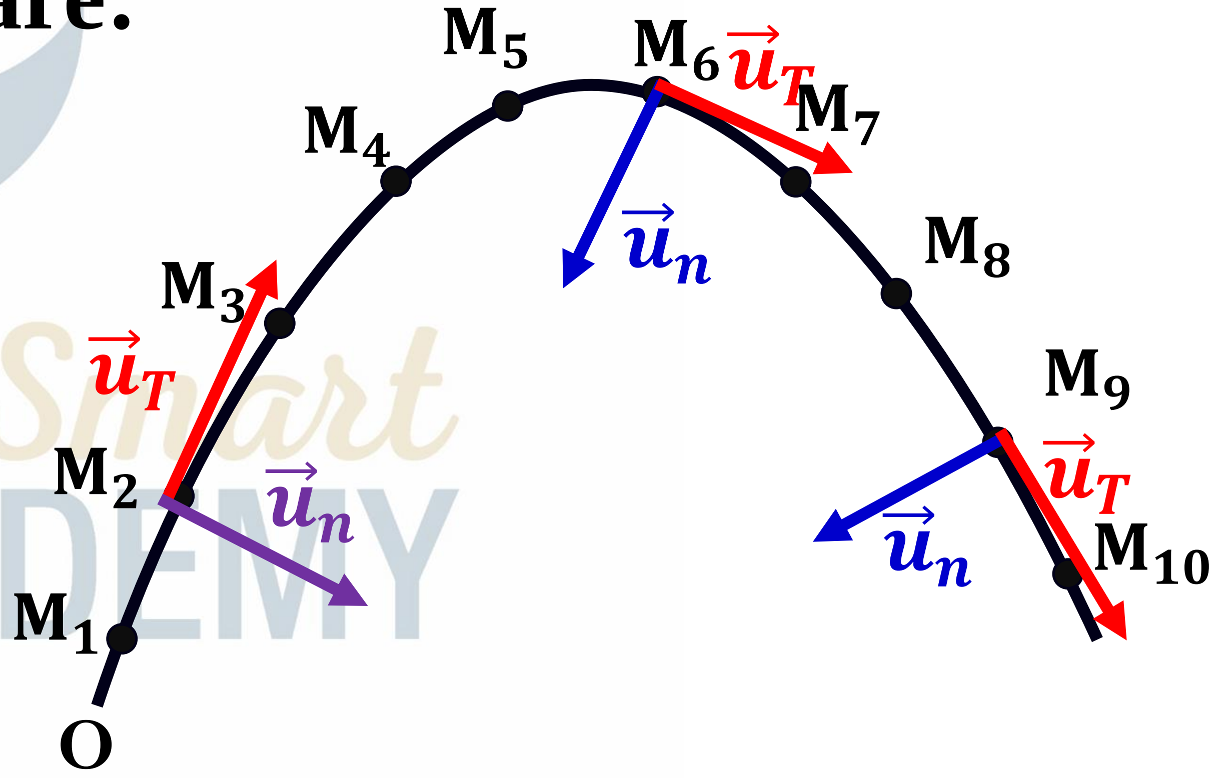
What is curvilinear system?

It is a system which moves with the particle. It is not fixed.

The unit vectors of this system are:

$\vec{u}_T$: Unit vector tangent to the trajectory and with the motion.

$\vec{u}_n$: Unit vector normal to the trajectory and directed towards the center of the trajectory

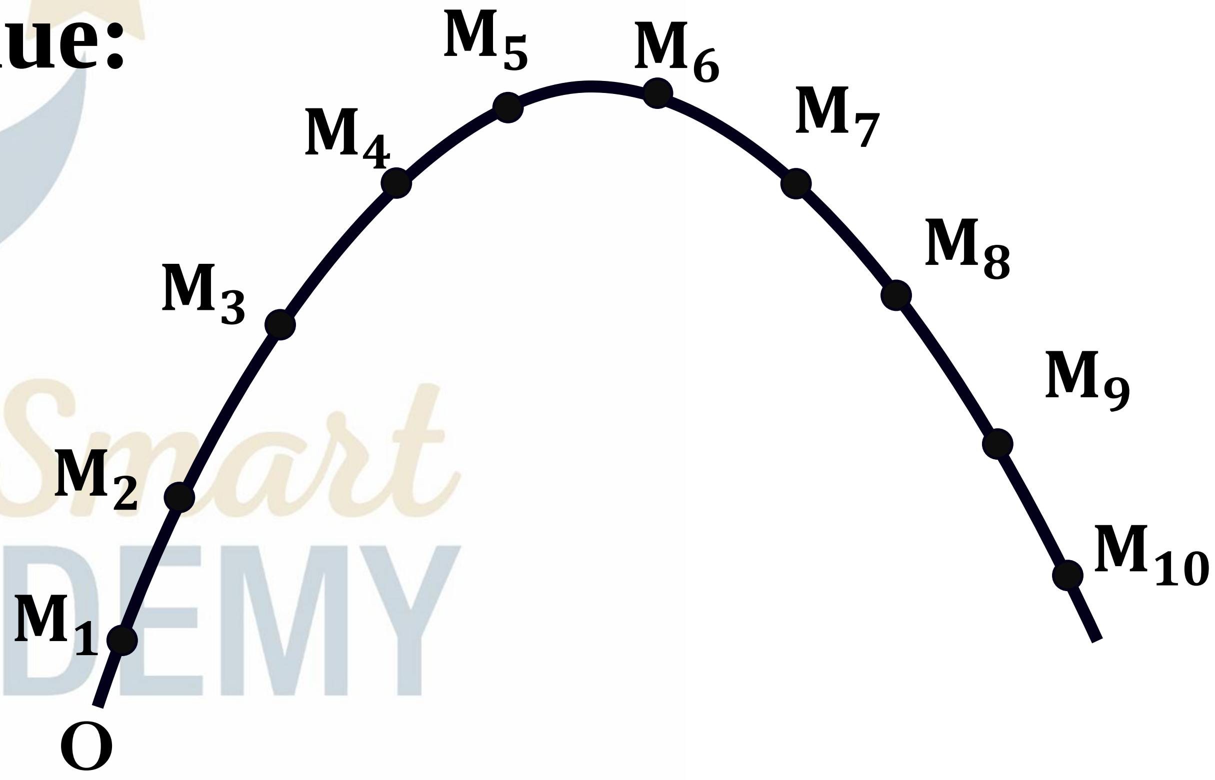


position and displacement in curvilinear system

Curvilinear abscissa (Position):

It is the position of the particle M at a certain instant t. It is determined by the algebraic value:

$$OM = s(t) \ in \ m \ SI$$



The position of M at t_2 is $OM_2 = s_2$

The position of M at t_5 is $OM_5 = s_5$

position and displacement in curvilinear system

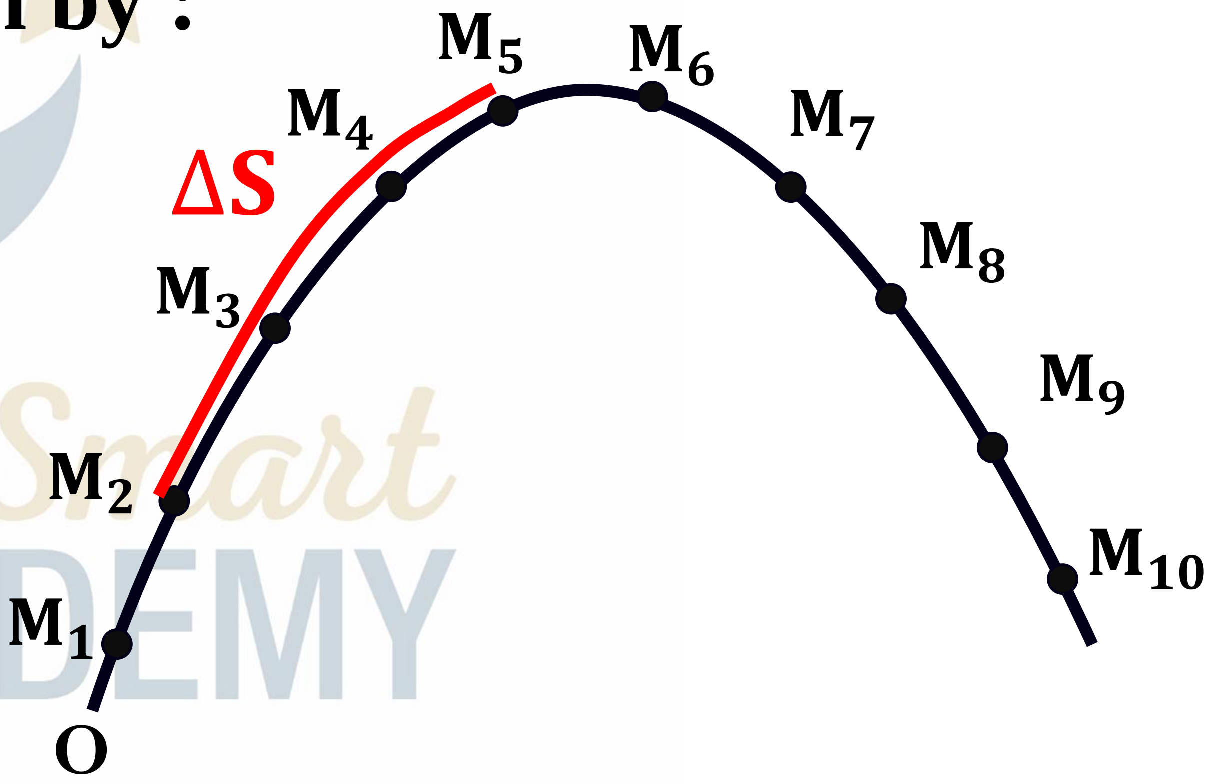
Displacement: The displacement of a particle M between two instants is determined by the variation of the curvilinear position. It is given by :

$$M_iM_f = \Delta s = s_f - s_i$$

When the particle moves from M_2 *to* M_5 hence the displacement is :

$$M_2M_5 = \Delta S = OM_5 - OM_2$$

$$\Delta s = s_5 - s_2$$



Speed and velocity vector in curvilinear system

The speed is the variation of the curvilinear position during a certain interval of time.

$$V = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i}$$

Instantaneous speed:

For a small interval of time $V = lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = lim_{t_f \to t_i} \frac{s_f - s_i}{t_f - t_i}$

$$V(t) = s'(t)$$

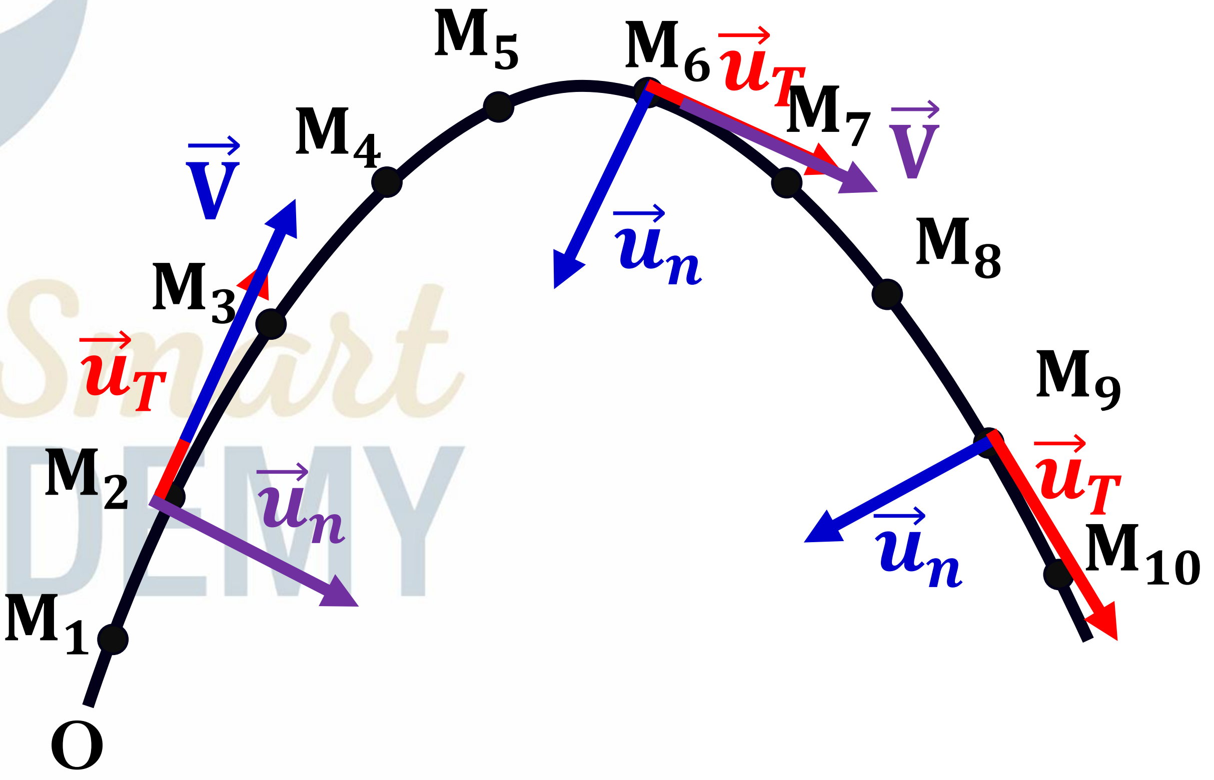
Speed and velocity vector in curvilinear system

Velocity vector:

The velocity vector is always tangent to the trajectory hence it is directed along the unit vector $\vec{u}_T$.

$$\vec{V} = V.\vec{u}_T$$

The velocity vector ($\vec{V}$) takes the direction of the tangential unit vector ($\vec{u}_T$)



Speed and velocity vector in curvilinear system

Application 13:

Consider a particle M moving along the curvilinear position $OM = s\,(t) = 2t^2 - t + 4$

1)Determine position of M at $t_0 = 0s$ and $t_1 = 1s$.

At $t_0 = 0s$: ⇨ $S_0 = 2(0)^2 - (0) + 4$ ⇨ $S_0 = 4\,m$

At $t_1 = 1s$, ⇨ $S_1 = 2(1)^2 - 1 + 4$ ⇨ $S_1 = 5\,m$

Speed and velocity vector in curvilinear system

2) **Determine the speed V at any instant and deduce the velocity vector $\vec{V}(t)$.**

The speed is the derivative of the curvilinear position

$$OM = s\,(t) = 2t^2 - t + 4$$

$$V(t) = s'(t) = 4t - 1$$

$$\vec{V} = V.\vec{u}_t = (4t - 1)\vec{u}_t$$

$$\vec{V} = (4t - 1)\vec{u}_t$$

Speed and velocity vector in curvilinear system

3) Deduce $\vec{V}_1$

$$\vec{V} = (4t - 1)\vec{u}_t$$

For $t_1 = 1s$

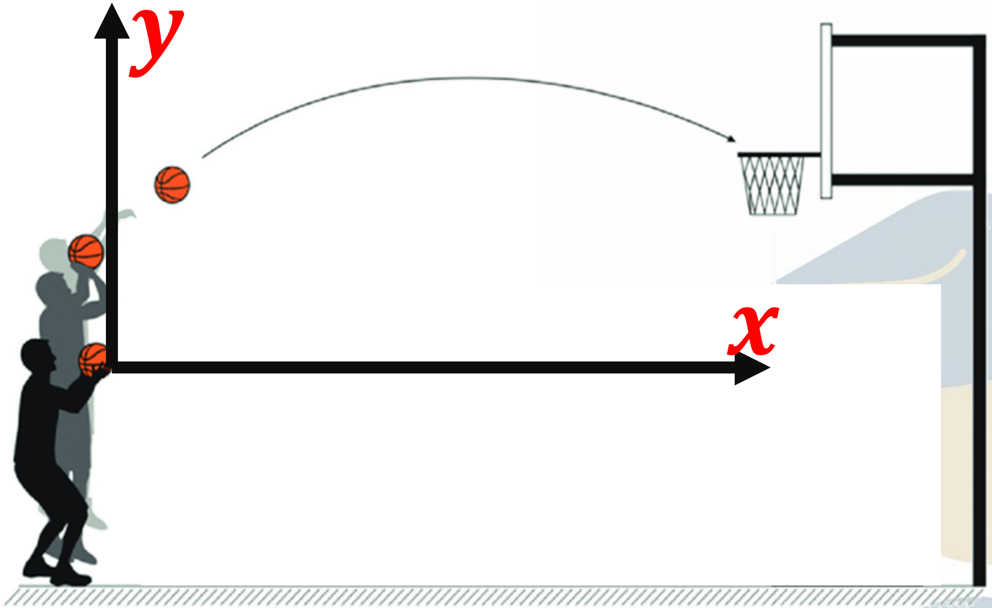
$$\vec{V}_1 = (4t - 1)\vec{u}_t$$

$$\vec{V}_1 = (4(1) - 1)\vec{u}_t$$

$$\vec{V}_1 = (4 - 1)\vec{u}_t$$

$$\vec{V}_1 = 3\vec{u}_t$$

The End



Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

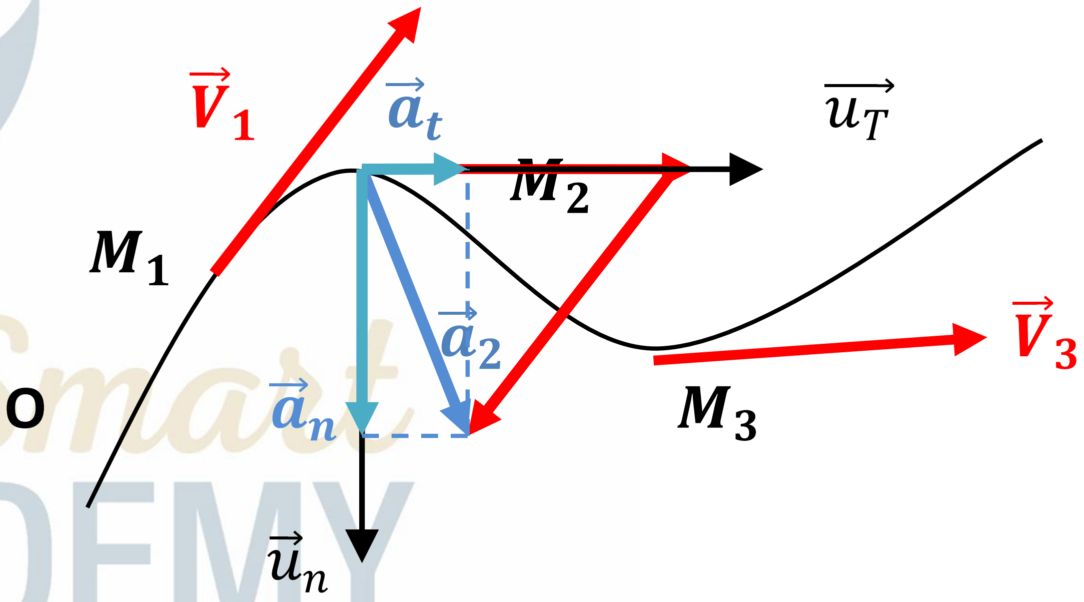
1. **Determine the acceleration and acceleration vector in curvilinear system**

Acceleration and acceleration vector in curvilinear system

The acceleration vector has two components, one along $\vec{u}_n$ and the other along $\vec{u}_T$.

The acceleration vector in this system is given by:

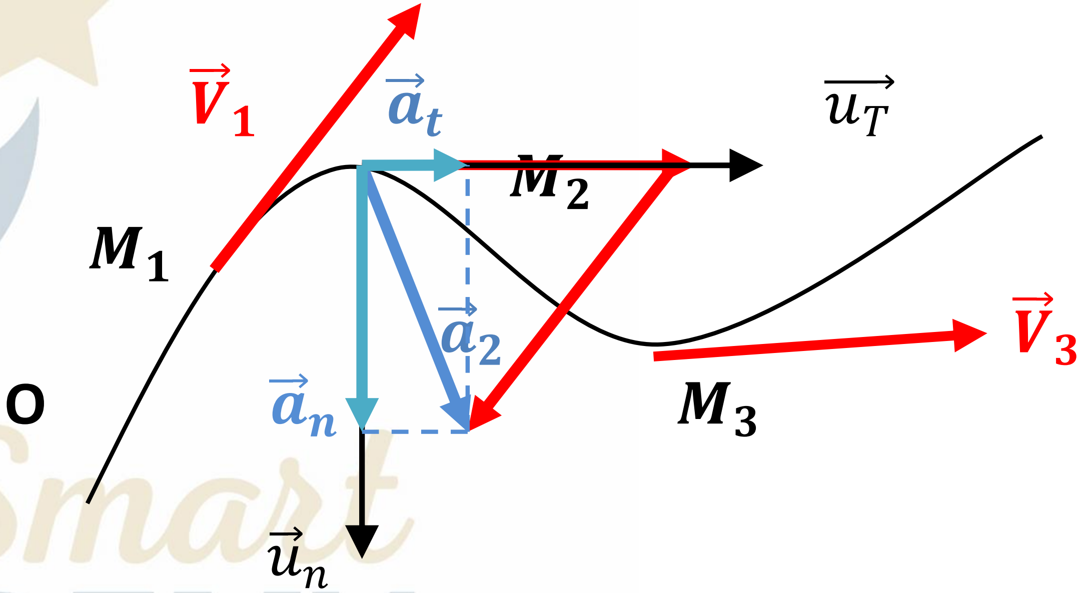
$$\vec{a} = a_n\vec{u}_n + a_T\vec{u}_T$$



Acceleration and acceleration vector in curvilinear system

$$\vec{a} = a_n\vec{u}_n + a_T\vec{u}_T$$

$$a_n = \frac{V^2}{R}$$



R : radius of curvature (m)

$$a_t = V'(\mathrm{t})$$

a_T is the derivative of the value (magnitude of the speed)

Acceleration and acceleration vector in curvilinear system

The value (magnitude) of the total acceleration (a) is the sum of the square of the two compoments a_T and a_n.

The value (magnitude) of the total acceleration (a) is given by:

$$a = \sqrt{a_n^2 + a_t^2}$$

All components and all forms of acceleration is expressed in m / s^2

Application 14:

Consider a particle M moving along the curvilinear position $OM = s\,(t) = 4t^2 - 2t + 4$

1)Determine the speed V at $t_1 = 1s$.

2)Determine the normal and tangential acceleration at $t_1 = 1s$, knowing that the radius of curvature is $R = 2m$ at this instant.

3) Deduce the value of the acceleration a_1 at this instant.

Speed and velocity vector in curvilinear system

$$OM = s\,(t) = 4t^2 - 2t + 4$$

1) Determine the speed V at $t_1 = 1s$.

The speed is the derivative of the curvilinear position

$$V(t) = s'(t) = 8t - 2$$

At $t_1 = 1s$:

$$V_1 = 8(1) - 2$$

$$V_1 = 8 - 2$$

$$V_1 = 6m/s$$

Speed and velocity vector in curvilinear system

$V(t) = 8t - 2; V_1 = 6m/s$

2) Determine the normal and tangential acceleration at $t_1 = 1s$, knowing that the radius $R = 2m$ at this instant.

$$a_n = \frac{V^2}{R}$$ ➡ For $t = 1s;\ V_1 = 6m/s$

Hence: $$a_n = \frac{V_1^2}{2}$$ ➡ $$a_n = \frac{(6)^2}{2} = \frac{36}{2}$$

$$a_n = 18\ \mathrm{m}/s^2$$

Speed and velocity vector in curvilinear system

$V(t) = 8t - 2; V_1 = 6m/s; a_n = 18m/s^2$

a_t is the derivative of speed

$a_T = V'$ ⇒ $a_t = 8m/s^2$

3) Deduce the value of the acceleration a_1 at this instant.

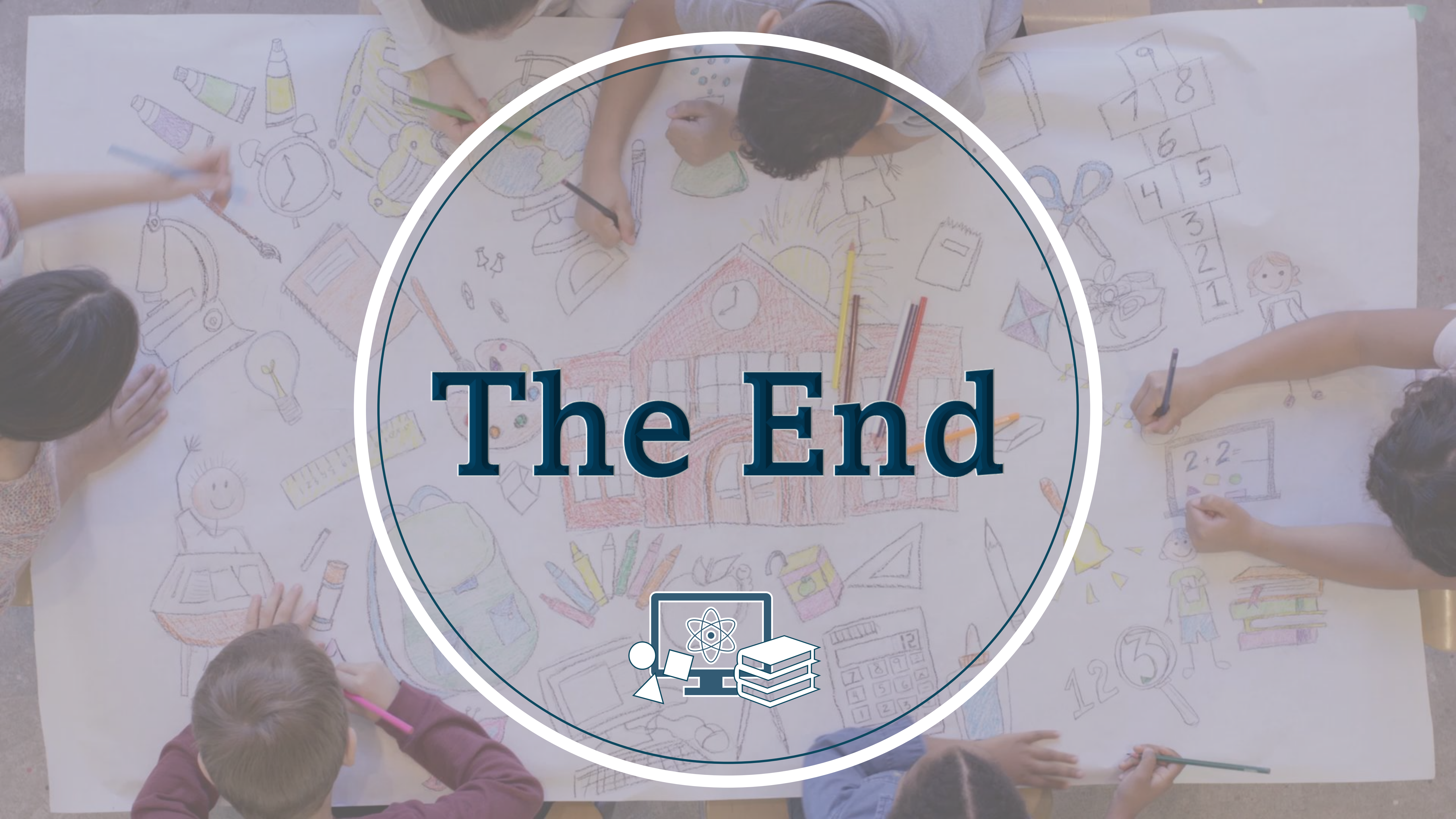
$$a_1^2 = a_n^2 + a_T^2$$

$$a_1 = \sqrt{a_n^2 + a_T^2}$$

$$a_1 = \sqrt{(18)^2 + (8)^2}$$

$$a_1 = \sqrt{324 + 64}$$

$$a_1 = 19.7m/s^2$$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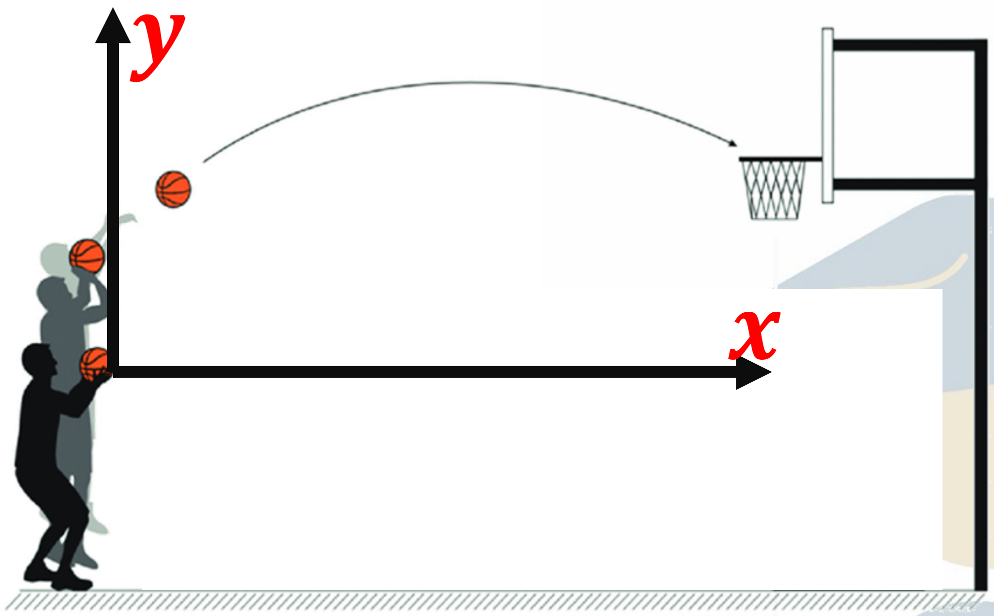


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

OBJECTIVES

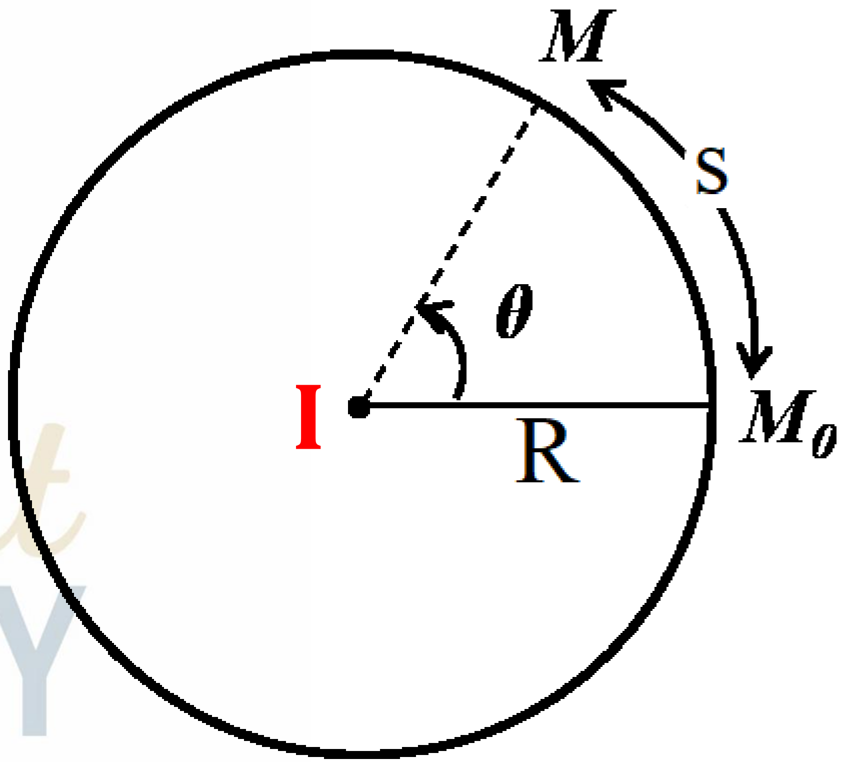
1. Determine the angular position (θ) in angular system.
2. Determine the angular speed in angular system.
3. Determine the angular acceleration in angular system.

Angular system (Circular motion)

Let M be a particle that moves along a circular path, where M_0 is the initial position and I is the center of the circle.

Angular position (θ):

The angular position of the particle M at a given time t is the angle θ (rd) Formed by the arc covered by the object.



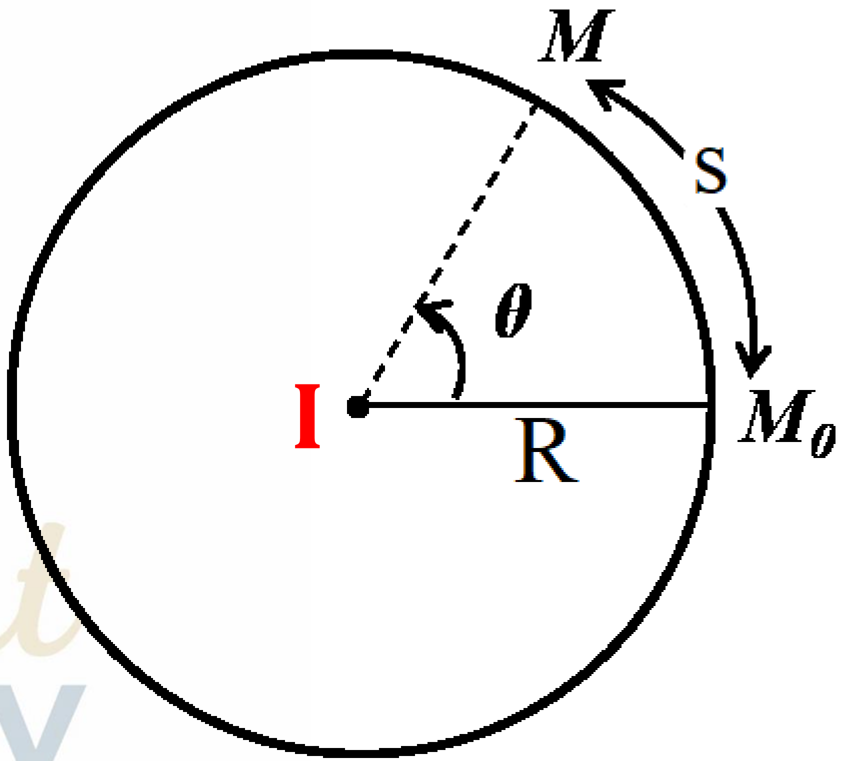
The angular abscissa (position) is:

$$\theta_{(rd)} = \frac{S_{(m)}}{R_{(m)}} \qquad \theta_{(rd)} = 2.\pi.n_{(rot)}$$

Where n is the number of rotation per time t

Angular system (Circular motion)

Angular velocity (ω): The angular velocity is the rate of variation of angle θ relative to time.



$$\omega_{(rd/s)} = \theta' = \frac{\Delta\theta}{\Delta t}$$

Where θ' is the derivative of angular position θ.

$$\omega_{(rd/s)} = \frac{V_{(m/s)}}{R_{(m)}}$$

$$\omega = 2.\pi.N$$

N or f: *frequency of rotation is the number of rotations per one second*

Angular system (Circular motion)

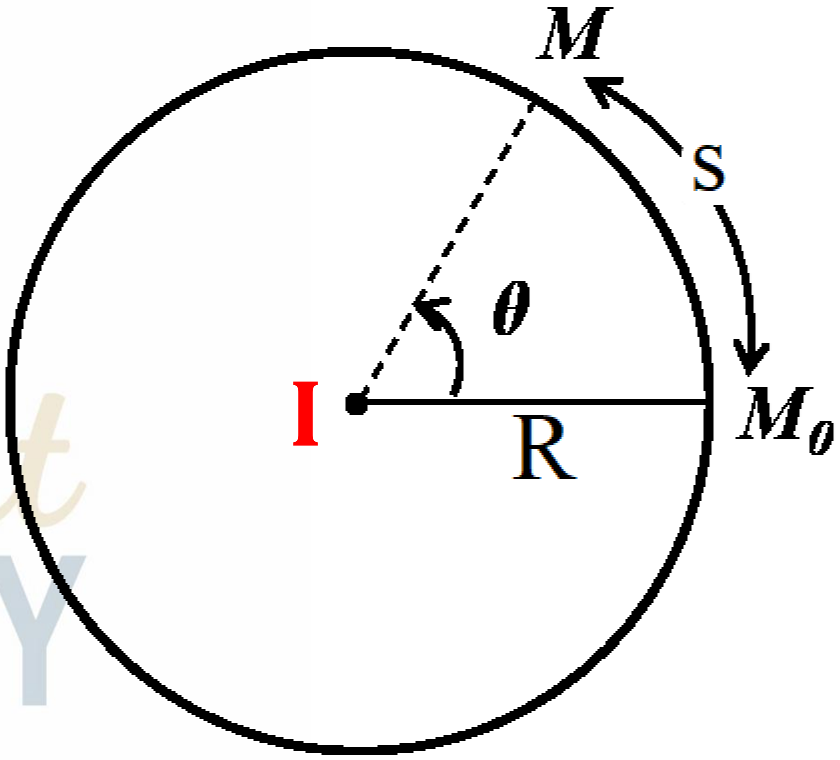
Angular acceleration (θ''): The angular acceleration is the rate of change of angular speed ω relative to time.

$$\theta'' = \omega' = \frac{\Delta\omega}{\Delta t}$$

θ'' is the derivative of angular speed

$$\theta'' = \left[\frac{V}{R}\right]' = \frac{V'}{R}$$

$$\theta'' = \frac{a_t}{R}$$

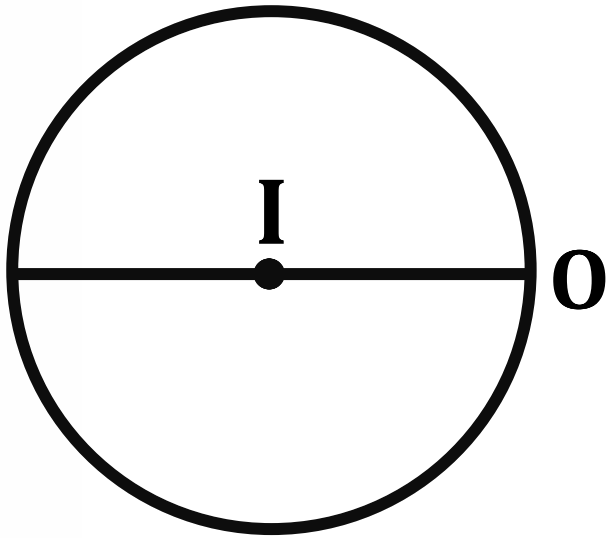


θ'' expressed in rd / s^2

Angular system (Circular motion)

Application 15: A mobile M describes a circumference of radius R = 2 m.

The time equation of the angular abscissa of the movement, relative to a given origin, is written by: $\theta(t) = 2t^2 + t + \frac{\pi}{4}$ in S.I



1) Verify that the mobile M is not located at O at $t = 0\ s$ and place M in the figure.
2) Determine the angular velocity of M as a function of time and deduce its value for $t = 1s$.
3) Determine the angular acceleration of M as a function of time.

Angular system (Circular motion)

$$\theta(t) = 2t^2 + t + \frac{\pi}{4}$$

1) Verify that the mobile M is not located at O at $t = 0\ s$ and place M in the figure.

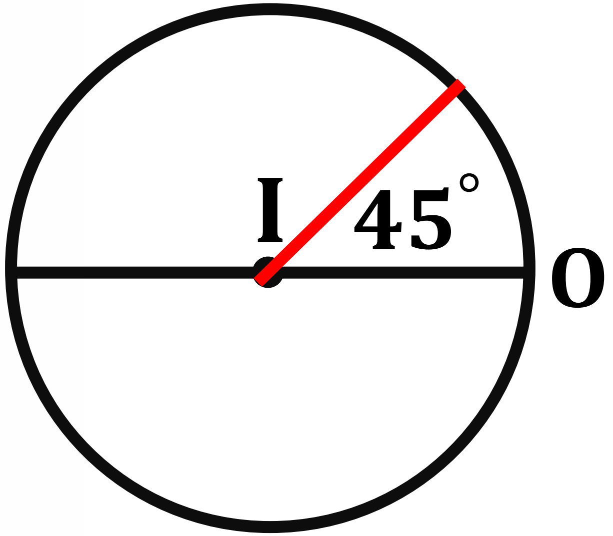
At $t = 0$: ➡ $\theta(0) = 2(0)^2 + (0) + \frac{\pi}{4}$ ➡ $\theta(0) = \frac{\pi}{4}$

M is not found in O

$\pi(rd) \rightarrow 180^o$

$\frac{\pi}{4} \rightarrow \theta_0 = ?$

$$\theta_0 = \frac{\frac{\pi}{4} \times 180}{\pi} = 45^o$$



Angular system (Circular motion)

$$\theta(t) = 2t^2 + t + \frac{\pi}{4}$$

2)Determine the angular velocity of M as a function of time and deduce its value for t = 1s.

The angular velocity of M is the derivative of the angular position.

$$\omega = \theta' = 4t + 1$$

at $t = 1s$;

$$\theta' = 4(1) + 1$$

$$\theta' = 5\, rd/s$$

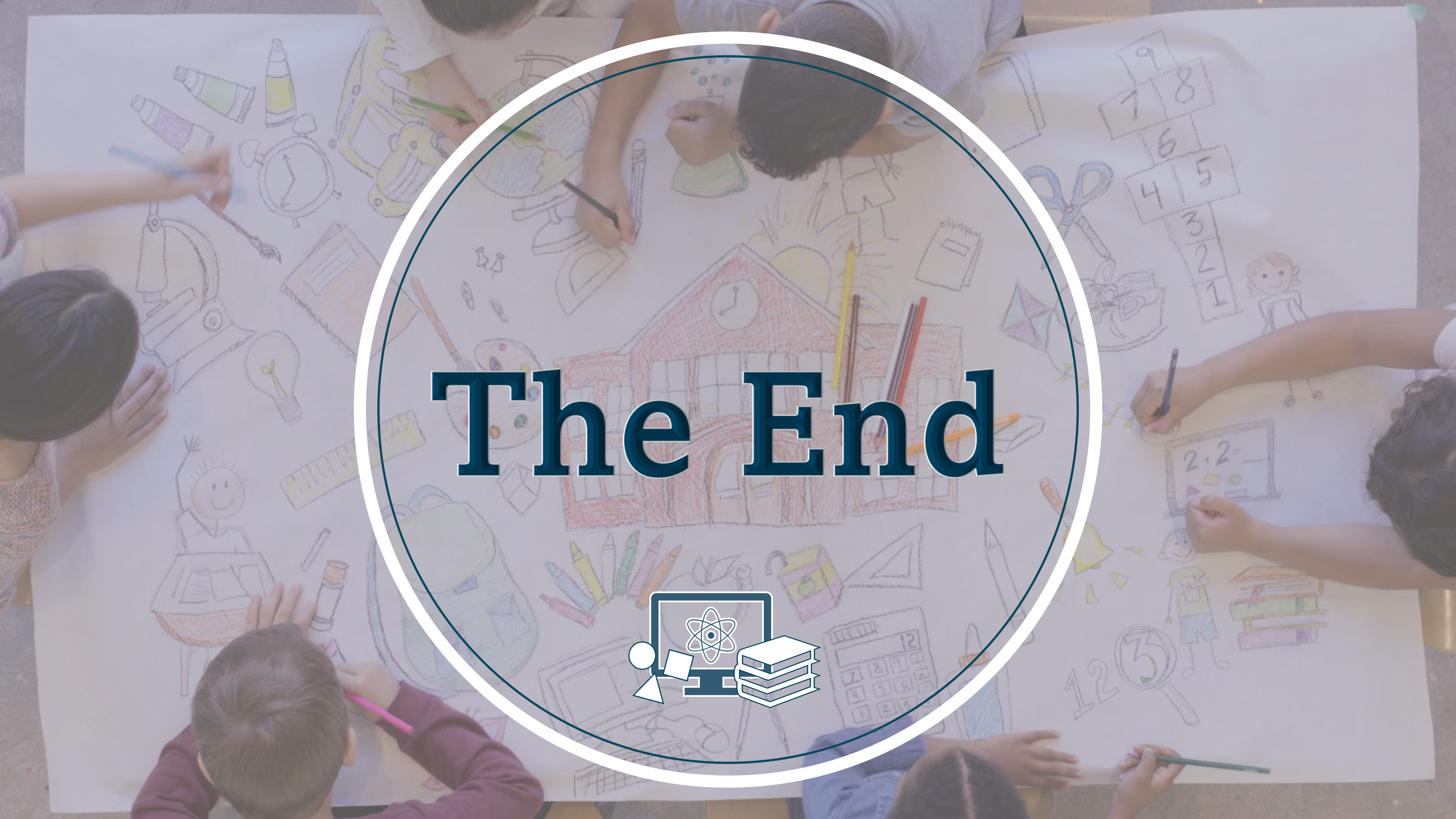
Angular system (Circular motion)

$$\theta(t) = 2t^2 + t + \frac{\pi}{4}; \ \theta' = 4t + 1$$

3) **Determine the angular acceleration of M as a function of time.**

The angular acceleration of M is the derivative of angular speed.

$$\theta'' = 4rd\ /\ s^2$$



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